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NGA STANDARDIZATION DOCUMENT

Accuracy and Predicted Accuracy in the NSG: Overview and Methodologies

Technical Guidance Document (TGD) 1

(2020-02-20)

Version 1.2

Forward

This handbook is approved for use by all Departments and Agencies of the Department of Defense.

Comments, suggestions, or questions on this document should be addressed to the GWG World Geodetic System (WGS) and Geomatics (WGSG) Focus Group, ATTN : Chair, WGS/Geomatics Standards Focus Group, ncgis-mail.nga.mil or to the National Geospatial-Intelligence Agency Office of Geomatics (SFN), Mail Stop L-41, 3838 Vogel Road, Arnold, MO 63010 or emailed to GandG@nga.mil.

Summary of Changes and Modifications

Revision	Date	Status	Description
Version 1.1	2018-02-27		Added overview and summary of TGD 2a through TGD 2f: section 4.7.1.
			Updated and additional descriptions of accuracy and predicted accuracy: sections 3.1.1, 3.1.11, 4.1.1, 4.1.2, 4.1.3, 4.6, 4.6.1, 4.6.2, 4.6.2.1, 4.6.3, Appendix B.
Version 1.2	2020-02-20		Minor modifications, additions, and reorganization throughout the document for improved readability.

			Moved content of previous section 4.6.2.1 and 4.6.3 to appendices B and C, respectively, for improved readability.
			Updated overview and summary of TGD 2a through TGD 2f based on their latest versions: section 4.7.1.
			A major addition to Section 5.1 corresponds to examples of the specification and validation of an accuracy requirement and a predicted accuracy requirement that were detailed in TGD 2c (Specification and Validation); related comments included in new Appendix D.
			Added an example of temporal correlation's effect on predicted accuracies in Section 5.3.3.
			Added Section 5.5.3 illustrating the use of error covariance matrices/ellipsoids for operational decisions and general analyses.
			Major additions to Section 5.9 correspond to an overview of estimators in general, and QA/QC based on measurement editing and the new and recommended confidence interval test for the reference variance as detailed in to TGD 2d (Estimators and their QC).
			Major additions to Section 5.11 correspond to examples extracted from TGD 2e (Monte-Carlo Simulation).
			Major additions to Section 5.12 correspond to examples extracted from TGD 2f (External Data and its Quality Assessment).
			More detail regarding different WGS 84 datums in Section 5.13 on provenance for predicted accuracy.

Contents

1 Scope

This Technical Guidance Document (TGD) 1 is the first in a series regarding Accuracy and Predicted Accuracy in the National System for Geospatial Intelligence (NSG). It is officially entitled "Accuracy and Predicted Accuracy in the NSG: Overview and Methodologies". As the title suggests, it includes an overview of the more detailed Technical Guidance Documents TGD 2a – TGD 2f listed below:

TGD 2a Accuracy and Predicted Accuracy in the NSG: Predictive Statistics

TGD 2b Accuracy and Predicted Accuracy in the NSG: Sample Statistics

TGD 2c Accuracy and Predicted Accuracy in the NSG: Specification and Validation

TGD 2d Accuracy and Predicted Accuracy in the NSG: Estimators and their Quality Control

TGD 2e Accuracy and Predicted Accuracy in the NSG: Monte-Carlo Simulation

TGD 2f Accuracy and Predicted Accuracy in the NSG: External Data and its Quality Assessment

The series is also supported by a compiled glossary of relevant terms:

TGD 1-G Accuracy and Predicted Accuracy in the NSG: Glossary of Terms

All documents in the series, "Accuracy and Predicted Accuracy in the NSG", are intended to provide technical guidance to inform the development of geospatial data accuracy characterization for NSG GEOINT collectors, producers and consumers -- accuracy characterization as required to describe the trustworthiness of geolocations for defense and intelligence use and to support practices that acquire, generate, process, exploit, and provide geolocation data and information based on geolocation data. Today, both the sources and desired uses for geospatial data are quickly expanding. Throughout the NSG, trusted conveyance of geospatial accuracy is broadly required for a variety of traditional and evolving missions including those supported by manual, man-in-the-loop, and automated processes. This guidance is the foundation layer for a collection of common techniques, methods, and algorithms ensuring that geospatial data within the NSG can be clearly requested, delivered and evaluated as fit for desired purpose whether by decision makers, intelligence analysts, or as input to further processing techniques.

TGD 1 contains references to and is referenced by all of the other more detailed Technical Guidance Documents. These documents, TGD 2a – TGD 2f, also have some cross-references among themselves. All Technical Guidance Documents also reference external public as well as "NGA approved for public release" documents for further insight/details. While each individual document contains definitions for important relevant terms, TGD 1-G compiles all important terms and respective definitions of use particular to this series of documents to ensure continuity and provide ease of reference.

The TGD 2 documents are also considered somewhat top-level in that they are not directed at specific systems. They do provide general guidance, technical insight, and recommended algorithms. The relationship of the Technical Guidance Documents with specific GEOINT Standards documents and specific Program Requirements documents is presented in Figure 1-1, where arrows refer to references. That is, in general, specific product requirement documents reference specific GEOINT standards documents which reference specific technical guidance documents.

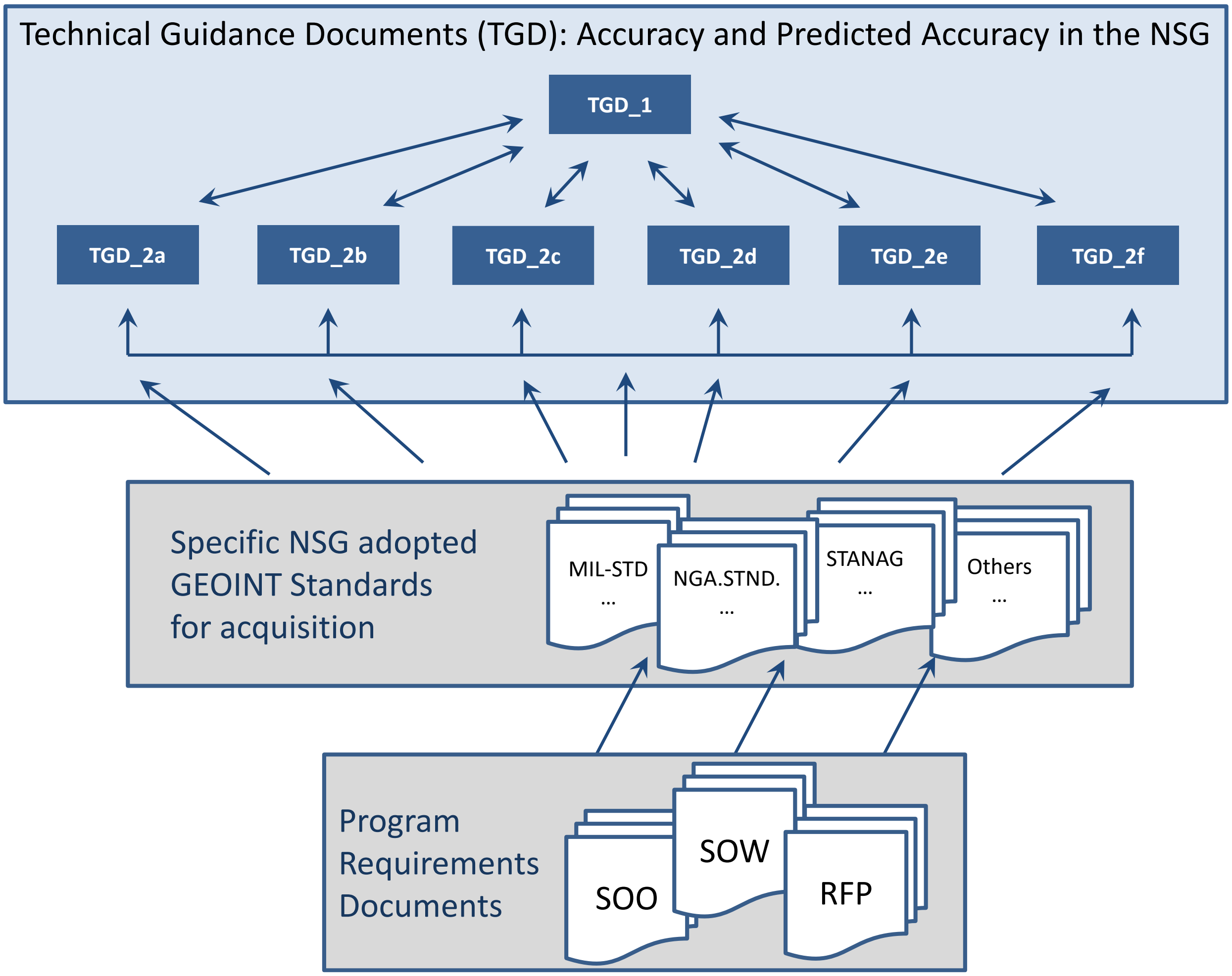


Figure 1-1: The relationships between the Technical Guidance Documents, GEOINT Standards Documents, and Program Requirement Documents

Accuracy and Predicted Accuracy in the NSG: Overview and Methodologies, Technical Guidance Document (TGD) 1 is for guidance only and cannot be cited as a requirement.

2 Applicable Documents

The documents listed below are not necessarily all of the documents referenced herein, but are those needed to understand the information provided by this information and guidance document.

2.1 Government specifications, standards, and handbooks

NGA.SIG.0026.02_1.1_ACCGLOS, Accuracy and Predicted Accuracy in the NSG: Glossary of Terms, Technical Guidance Document (TGD) 1-G

NGA.SIG.0026.03_1.1_ACCPRED, Accuracy and Predicted Accuracy in the NSG: Predictive Statistics, Technical Guidance Document (TGD) 2a

NGA.SIG.0026.04_1.0_ACCSAMP, Accuracy and Predicted Accuracy in the NSG: Sample Statistics, Technical Guidance Document (TGD) 2b

NGA.SIG.0026.05_1.1_ACCSPEC, Accuracy and Predicted Accuracy in the NSG: Specification and Validation, Technical Guidance Document (TGD) 2c

NGA.SIG.0026.06_1.0_ACCESQC, Accuracy and Predicted Accuracy in the NSG: Estimators and their Quality Control, Technical Guidance Document (TGD) 2d

NGA.SIG.0026.07_1.0_ACCMTCO, Accuracy and Predicted Accuracy in the NSG: Monte-Carlo Simulation, Technical Guidance Document (TGD) 2e

NGA.SIG.0026.08_1.0_ACCXDQA, Accuracy and Predicted Accuracy in the NSG: External Data and its Quality Assessment, Technical Guidance Document (TGD) 2f

3 Definitions

There are a number of authoritative guides as well as existing standards within the NSG and Department of Defense for definitions of the identified key terms used in this technical guidance document. In many cases, the existing definitions provided by these sources are either too general or, in some cases, too narrow or dated by intended purposes contemporary to the document's development and publication. The definitions provided in this document have been expanded and refined to explicitly address details relevant to the current and desired future use of accuracy in the NSG. To acknowledge the basis and/or linage of certain terms Section 3.1, we reference the following sources considered as either foundational or contributory:

[a] Anderson, James M. and Mikhail, E., Surveying: Theory and Practice, 7th Edition, WCB/McGraw-Hill, 1998.

[b] DMA-TR-8400.1, DMA Technical Report: Error Theory as Applied to Mapping, Charting, and Geodesy.

[c] Defense Mapping Agency, Glossary of Mapping, Charting, and Geodetic Terms, 4th Edition, Defense Mapping Agency Hydrographic/Topographic Center, 1981.

[d] ISO TC/211 211n2047, Text for ISO 19111 Geographic Information - Spatial referencing by coordinates, as sent to the ISO Central Secretariat for issuing as FDIS, July 17, 2006.

[e] Joint Publication (JP) 1-02, Department of Defense Dictionary of Military and Associated Terms, November 8, 2010 as amended through January 15, 2016.

[f] MIL-HDBK-850, Military Handbook: Glossary of Mapping, Charting, and Geodetic Terms, January 21, 1994.

[g] MIL-STD-2401, Department of Defense Standard Practice; Department of Defense World Geodetic System (WGS), January 11, 1994

[h] MIL-STD-600001, Department of Defense Standard Practice; Mapping, Charting and Geodesy Accuracy, February 26, 1990.

[i] *National System for Geospatial Intelligence* [Brochure] Public Release Case #15-489.

[j] NGA.STND.0046_1.0, The Generic Point-cloud Model (GPM): Implementation and Exploitation, Version 1.0, October 03, 2015.

[k] Oxford Dictionaries (www.oxforddictionaries.com/us/) copyright © 2016 by Oxford University Press.

[l] Soler, Tomas and Hothem, L., “Coordinate Systems Used in Geodesy: Basic Definitions and Concepts”, Journal of Surveying Engineering, Vol. 114, No. 2, May 1988.

3.1 Key Terms Used in the Document

3.1.1 Accuracy

The range of values for the error in an object’s metric value with respect to an accepted reference value expressed as a probability.

- Statements of accuracy may be developed through applications of predictive statistics or by sample statistics based on multiple independent samples of errors. [f]

See Appendix A for a more detailed and augmented definition.

3.1.2 Circular Error

See "Section 3.1.17 Scalar Accuracy Metrics".

3.1.3 Crowd-sourcing

The process of obtaining data, in particular geospatial data, via individual contributions from a large group of people such as an online community, typically on a volunteered basis.

3.1.4 Error

The difference between the observed or estimated value and its ideal or true value. See Appendix A for a more detailed and augmented definition. [f]

3.1.5 External Data

In the context of this document, External Data is geospatial data that is obtained by purchase or openly available public sources. Outsourced data, commodities data, and crowd-sourced data are examples of External Data.

3.1.6 Fusion

A process that combines or relates different sources of (typically independent) information.

3.1.7 Linear Error

See "Section 3.1.17 Scalar Accuracy Metrics".

3.1.8 Monte-Carlo Simulation

A technique in which a large number of independent sample inputs for a system are randomly generated using an assumed *a priori* statistical model to analyze corresponding system output samples statistically and support derivation of a statistical model of the system output. This technique is valuable for complex systems, non-linear systems, and those where no insight to internal algorithms is provided ("black box" systems).

3.1.9 National System for Geospatial Intelligence (NSG)

The operating framework supported by producers, consumers or influencers of geospatial intelligence (GEOINT). Spanning defense, intelligence, civil, commercial, academic and international sectors, the NSG contributes to the overall advancement of the GEOINT function within the strategic priorities identified by the Functional Manager for Geospatial Intelligence in the role established by Executive Order 12333. The framework facilitates community strategy, policy, governance, standards and requirements to ensure responsive, integrated national security capabilities. [i]

3.1.10 Outsourced Data

Data through purchase (contract) which may be contingent on specified collection or production criteria.

3.1.11 Predicted Accuracy

The range of values for the error in a specific object's metric value expressed as a probability derived from an underlying and accompanying detailed statistical error model.

- If the statistical error model does not include the identification of a specific probability distribution, a Gaussian (or Normal) probability distribution is typically assumed in order to generate probabilities.
- The term "Predicted" in Predicted Accuracy corresponds to the use of predictive statistics in the detailed statistical error model; it does not correspond to a prediction of accuracy applicable to the future since the corresponding error corresponds to a geolocation already extracted.

See Appendix A for a more detailed and augmented definition.

3.1.12 Predictive Statistics

Statistics corresponding to the mathematical modeling of assumed *a pri*ori error characteristics contained in a statistical error model.

3.1.13 Quality Assurance

The maintenance of a desired level of quality in a service or product, especially by means of attention to every stage of the process of delivery or production. [k]

- For Estimators in the NSG, Quality Assurance (QA) corresponds to the requirement to embed the generation of various statistics, analyses, and related procedures in the overall solution process which insure the validity (reliability) of the estimators solution X and its error covariance matrix C_X.

3.1.14 Quality Assessment

Processes and procedures intended to verify the reliability of provided data and processes, typically performed independent of collection or production. For example, If ground truth is available, then comparison of actual (sample) errors to predicted errors (statistical values via rigorous error propagation) is a key part of this process.

3.1.15 Quality Control

For Estimators in the NSG, Quality Control (QC) corresponds to implementation of a QA requirement to embed the generation of various statistics, analyses, and related procedures in the overall solution process such that the quality (reliability) of the specific solution is assured.

3.1.16 Sample Statistics

Statistics corresponding to the analysis of a collection of physical observations, a sample of the population, as compared to an assumed true or an *a priori* value.

3.1.17 Scalar Accuracy Metrics

Convenient one-number summaries of geolocation accuracy and geolocation predicted accuracy expressed as a probability: (1) Linear Error (LE) or LE90 corresponds to 90% probable vertical error, (2) Circular Error (CE) or CE90 correspond to 90% probable horizontal radial error, and (3) Spherical Error (SE) or SE90 corresponds to 90% spherical radial error. [b],[f], and [h] See Appendix A for a more detailed and augmented definition.

3.1.18 Spherical Error
See "Section 3.1.17 Scalar Accuracy Metrics".

3.1.19 Statistical Error Model
Information which describes the error data corresponding to a given state vector. The information includes the type of corresponding error representation (random variable, random vector, stochastic process, or random process), the category of statistics (predictive or sample), and associated statistical information including at a minimum the mean-value and covariance data.

3.1.20 Validation
The process of determining the degree to which a model is an accurate representation of the real world from the perspective of its intended use/s. In the NSG, this includes validation of accuracy and predicted accuracy specified capabilities. [e]

3.1.21 Variance
The measure of the dispersion of a random variable about its mean-value, also the standard deviation squared. [b]

3.1.22 Verification
The process of determining that an implemented model accurately represents the developer's conceptual description and specifications. [e]

3.2 Other Relevant Terms
Appendix A contains definitions of the following additional terms relevant to the content of this document:

- *A priori*
- *A posteriori*
- Accuracy (augmented definition)
- Absolute Horizontal Accuracy
- Absolute Vertical Accuracy
- Accuracy Assessment Model
- Bias Error
- CE-LE Error Cylinder
- Confidence Ellipsoid
- Correlated Error
- Correlated Values
- Covariance
- Covariance Function
- Covariance Matrix
- Cross-covariance Matrix
- Deterministic Error
- Directed Percentile
- Earth Centered Earth Fixed Cartesian Coordinate System
- Elevation
- Error (augmented definition)
- Error Ellipsoid
- Estimator
- Gaussian (or Normal) probability distribution
- Geodetic Coordinate System
- Ground Truth
- Homogeneous
- Horizontal Error
- Inter-state vector correlation
- Intra-state vector correlation
- Local Tangent Plane Coordinate System
- Mean-Value
- Metadata

- Multi-Image Geopositioning (MIG)
- Multi-State Vector Error Covariance Matrix
- Order Statistics
- Percentile
- Precision
- Predicted Accuracy (augmented definition)
- Predicted Accuracy Model
- Principal Matrix Square Root
- Probability density function (pdf)
- Probability distribution
- Probability distribution function (cdf)
- Provenance
- Radial Error
- Random Error
- Random Field
- Random Variable
- Random Vector
- Realization
- Relative Horizontal Accuracy
- Relative Vertical Accuracy
- Rigorous Error Propagation
- Scalar Accuracy Metrics (augmented definition)
- Sensor support data (aka image metadata)
- Spatial Correlation
- Standard Deviation
- State Vector
- State Vector Error
- Stationary
- Stochastic Process
- Strictly Positive Definite Correlation Function
- Systematic Error
- Temporal Correlation
- Time Constant
- Uncertainty
- Uncorrelated Error
- Uncorrelated Values
- Vertical Error
- WGS 84

3.3 Abbreviations and Acronyms

Abbreviation/Acronym	Definition
1d	One Dimensional
2d	Two Dimensional
3d	Three Dimensional
API	Applications Program Interface
CE	Circular Error
DEM	Digital Elevation Model
DoD	U.S. Department of Defense
DSM	Digital Surface Model
DTED	Digital Terrain Elevation Data
ECF	Earth Centered Fixed
ENU	East North Up
EO	Electro-optical
GEOINT	Geospatial Intelligence
GPS	Global Positioning System
i.i.d.	independent and identically distributed
LE	Linear Error
LOS	Line-of-sight
MIG	Multi-Image Geopositioning

NSG	National System for Geospatial Intelligence
Pdcf	positive definite correlation function
QA	Quality Assurance
QC	Quality Control
RF	Random Field
RV	Random Vector
SAR	Synthetic Aperture Radar
SE	Spherical Error
SP	Stochastic Process
Spdcf	strictly positive definite correlation function
TC	Time Constant
TGD	Technical Guidance Document
WLS	Weighted Least Squares

4 Overview of Accuracy and Predicted Accuracy in the NSG

This level 1 Technical Guidance Document (TGD 1) presents a general introduction to accuracy and its role in the NSG. Recommended methodologies, procedures, and algorithms are introduced in an integrated but somewhat informal fashion. Other level 2 Technical Guidance Documents (TGD 2a – 2f) present corresponding details and are both summarized and referenced by this document.

4.1 The NSG and Accuracy: Depiction of a generic NSG Geolocation System

Accuracy and its proper representation play a vital role in the NSG; in particular, for the generic system represented by up to three major processes or modules and their representative states S as illustrated in Figure 4.1-1.

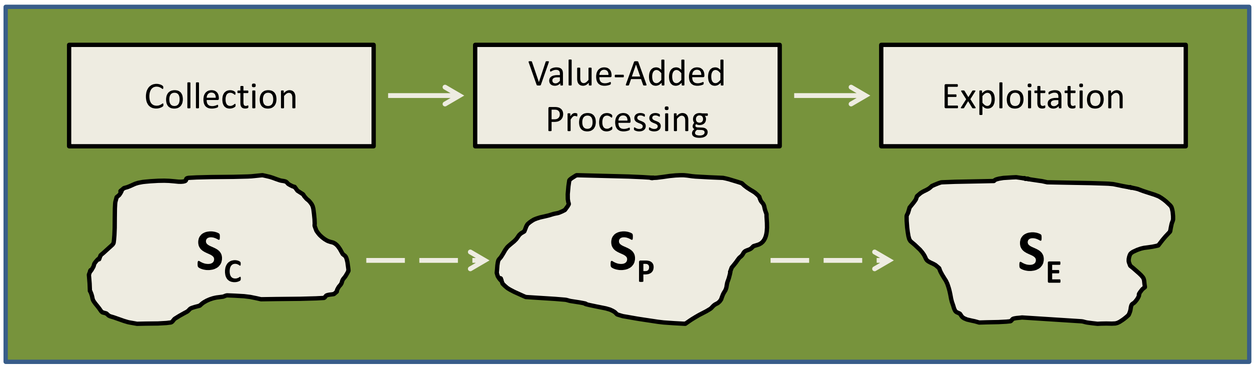


Figure 4.1-1: Major Processes (Modules) of an NSG Geolocation System

We are interested in the accuracy of an arbitrary geospatial "object" associated with the above NSG Geolocation System, whether the object is within one specific module or an input/output between modules. Relevant objects either affect geolocations that are produced or extracted by the system or are the geolocations themselves. For example, the geolocation of a "target" (or feature) generated by the Exploitation Module using data collected and processed by the Collection Module and possibly improved (corrected) by the Value-Added Processing Module.

As such, the Technical Guidance documents present recommended methods, procedures, and algorithms that ensure the best possible geolocation accuracies in the above system, including its various products, with corresponding reliable representations of those accuracies. The Technical Guidance documents address a wide range of geolocation-related activities, including: (1) the extraction or estimation of geolocations and their Quality Control, (2) the specification, validation, and general assessment of geolocation accuracy, (3) the supporting use of predictive as well as sample statistics, and (4) the use of Monte Carlo simulation in error modeling and product generation. Correspondingly, the actual definitions of accuracy and related quantities are important and defined as follows:

4.1.1 Accuracy and Predicted Accuracy

A common dictionary definition for "accuracy" is the degree to which something is true or exact. However, this definition is too limited for the NSG. We expand this general definition and define accuracy as follows for an NSG Geospatial System, and assume for now that geolocations are the objects of interest and relative to a specified geodetic reference system:

- **Accuracy**
  - The probability of error corresponding to an arbitrary 3d geolocation extracted by the system. The probability of error is typically expressed as CE90=XX meters, the 90% probability that horizontal circular or radial error is less than XX meters, as well as LE90=YY meters, the 90% probability that vertical linear error is less than YY meters. In general, the error is represented as a 3d random vector and its corresponding CE90 and LE90 values are typically specified and/or evaluated based on sample statistics of independent samples of error.

By itself, the above definition is still too limited. Therefore, we introduce the concept of "predicted accuracy", defined as follows:

- **Predicted accuracy**
  - A statistical description of the error in a specific geolocation extracted by the system. The error is expressed as a 3d random vector and the statistical description consists primarily of an error covariance matrix of the random vector about a mean-value typically assumed equal to zero unless specifically stated otherwise. The probability of error can also be computed if either a probability distribution is also specified or a multi-variate Gaussian probability distribution of error is assumed. The probability of error is expressed as a probability or confidence ellipsoid at a specified probability or confidence level, respectively, and may also be expressed as CE90 and LE90.

4.1.2 Specific geolocation and its predicted accuracy

A specific geolocation and its predicted accuracy are typically the output of an estimator within the Exploitation Module, such as a Weighted Least Squares (WLS) batch estimator or a Kalman Filter (KF) sequential estimator. The estimator actually estimates a $3x1$ state vector containing the 3d geolocation's coordinates using sensor-based measurements related to the geolocation. These measurements contain random errors; hence, the solution's state vector contains random errors as well that correspond to a $3x1$ random vector. This random vector is described by predictive statistics (predicted accuracy), primarily an $3x3$ error covariance matrix which may be used to generate corresponding probabilities of solution error.

The estimator's modeling of measurement errors and their effect on its solution and corresponding predicted accuracy are based on statistical error models and rigorous error propagation for (near) optimal solutions and reliable predicted accuracies.

4.1.3 Summary of an NSG Geolocations System's use of accuracy and predicted accuracy

Accuracy is used to describe the performance of an NSG Geolocation System, and in particular, is used to specify corresponding accuracy requirements for an arbitrary geolocation extracted by the system.

Predicted accuracy is generated for each specific geolocation extracted by the system and generally varies across them.

In addition, (near) optimal estimates of specific geolocations and corresponding reliable predicted accuracies require the use of proper statistical error models, both within the estimator and within the NSG Geolocation System in general, as further described in both this document and the level 2 Technical Guidance Documents.

Without the use of proper statistical error models and corresponding predictive statistics (aka predicted accuracy) throughout the NSG Geolocation System, system performance will be far from optimal or reliable – various information and data that affect final outputs or products cannot be combined properly. In particular, Exploitation cannot be optimal nor include reliable predicted accuracies of results. For the extraction of the 3d geolocation of a specific target of interest, corresponding geolocation errors will not be the smallest possible and their predicted accuracies will not be "tailored" to this specific target. Reliable predicted accuracies, tailored to the specific target of interest, are required for actionable intelligence, among other things.

4.2 Guide to the remaining sections in the Section 4 overview

Now that a depiction of a generic NSG Geospatial System has been presented along with a description of accuracy and predicted accuracy for context, an overview of the contents of the remaining sections in Section 4 follows:

Section 4.3 of this document presents a conceptual description of the state S of a Major Module in an NSG Geolocation System (Figure 4.1-1), which includes statistical error models. Section 4.4 presents various examples of NSG Geolocation Systems and their major modules. Section 4.5 discusses appropriate coordinate systems for use in an NSG Geolocation System.

The differences between accuracy and predicted accuracy for geolocations are further illustrated by example in Section 4.6, which also provides additional information regarding both. More specifically, Section 4.6.1 presents an example based on a Geolocation System that corresponds to a commercial satellite-based imaging system. Its Exploitation Module extracts 3d geolocations.

Section 4.6.2 then goes on to present an example that is focused on the predicted accuracy of an arbitrary but specific geolocation associated with a generic Geolocation System, but at a level "deeper" than that which was provided in Section 4.6.1 for a commercial satellite-based imaging system. It also further illustrates the relationship between predicted accuracy and the accuracy of the Geolocation System.

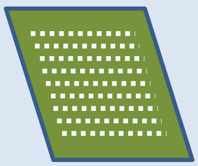
Section 4.7 presents a summary of the detailed TGD 2 documents as well as the remainder of this TGD 1 document. In particular, Section 4.7.1 presents an overview of the inter-relationships between the various TGD 2 documents and the contents of each. Section 4.7.2 presents a summary of the contents of the various sections that make-up the more detailed Section 5 of this document.

4.3 Representative State of a Geolocation System Major Module

Figure 4.3-1 presents a conceptual description of the top-level contents of the state S of a Major Module in an NSG Geolocation System. It consists of: (1) data, (2) a state vector describing important aspects of the data or containing estimates related to the data, and (3) a detailed statistical error model for the state vector (error), generally associated with its "predicted accuracy".

State **S**:

Data



State Vector

X

Statistical Error Model

Referenced but unknown error:

$\epsilon X \equiv X - X_{true}$

Provided descriptors of error:

Underlying Type: {RV, SP, RF}

Stats Category: {Predictive, Sample}

Stats: $\{\overline{\epsilon X}, C_{\epsilon X}, pdf_{\epsilon X}, \rho(q)\}$

Figure 4.3-1: Description of the top level contents of a module's state S

Although not shown explicitly in the above figure, a module's State S may consist of multiple sub-States S_i, $i = 1, .., k$, each containing data, state vectors, and error models. For example, S_1 could include state vector estimates of geolocations, and S_2 could include a state vector estimate of sensor metadata used to estimate geolocations. Corresponding statistical error models or predicted accuracy are included with each state vector estimate in both S_1 and in S_2. If the state vector estimates in S_1 were generated using sensor-based images, the data in S_1 would typically include the images, or at least image patches containing relevant image measurements.

<u>Statistical Error Model Content</u>

The statistical error model of Figure 4.3-1 is described in Section 5.2, and includes the identification of the underlying type of representation of the error: Random vector (RV), Stochastic process (SP), or Random Field (RF). The latter two representations of error correspond to a collection of random vectors parameterized by time and spatial location, respectively. A $nx1$ random vector contains n random variables as components.

A simple example of the representation of error is a single $3x1$ random vector ϵX that corresponds to the error in an estimator's solution or state vector X of a 3d geolocation. Another example is a collection of $3x1$ random vectors ϵX_i that correspond to the errors in a time sequence of sensor position (or attitude) X_i corresponding to a platform's trajectory. The X_i are part of the sensor's metadata, and the closer in time a pair X_i and X_j are, the greater the temporal correlation or statistical similarity of their errors ϵX_i and ϵX_j. A detailed overview of random variables, random vectors, stochastic processes, and random fields is presented in Section 5.3, with Section 5.3.1 containing graphic-based examples.

Also, with regards to statistical error models, the corresponding state vector X is very general. Instead of a geolocation or sensor metadata, for example, it can correspond to a vector of sensor-based measurements related to one or more geolocations. In this case, the random vector ϵX corresponds to a vector of measurement errors.

Statistics for an error represented as a random vector also include its $nx1$ mean-value $\overline{\epsilon X}$ and its nxn covariance matrix $C_{\epsilon X}$. From these statistics, the probability of error can also be derived, if so desired, assuming a Gaussian probability distribution of error. The latter assumption is not required if statistics are sample-based instead of predictive-based, or if the optional probability density function $pdf_{\epsilon X}$ is provided. Statistics may also include a strictly positive definite correlation function (spdcf) $\rho(q)$ which is used to represent the correlation (of errors) between a collection of random vectors represented as either a stochastic process or a random field.

4.4 Examples of NSG Geolocation Systems and their Major Modules

The NSG is large in scope. In order to give further insight into what constitutes one of its generic systems and its modules, two specific, but still somewhat generic, examples are as follows:

(1) An Imaging System:

- Collection Module: satellite-based imaging sensors and ground station to produce images and estimates of their original (*a priori*) metadata (sensor position, attitude, etc.) needed for the image-to-ground relationship; the ground station includes Kalman Filter/smoother estimators to generate the estimates and their predicted accuracies.
- Value-Added Processing Module (optional): Adjustment of the *a priori* metadata for improved predicted accuracy, typically using a batch Weighted Least Squares (WLS) estimator and based on information from related images and/or ground control; corresponding possible output (products) include:
    - Adjusted or *a posteriori* metadata (and imagery)
- Exploitation Module: the extraction of feature ("target") 3d locations from measurements in the images and corresponding predicted accuracy of the locations based on the above. The optimal extraction of 3d geolocations and corresponding predicted accuracy is termed Multi-Image Geopositioning (MIG), typically performed using a WLS estimator (see Section 5.9.2 for more detail). Possible products are based on the extraction of the geolocations of specific features and/or geolocations across a grid:
    - Digital Point Positioning Data Base (DPPDB)
    - Digital Terrain Elevation Data (DTED)
    - Digital Surface Model (DSM)

  Note that the above products could alternately be considered generated as part of a Value-Added Processing Module.

(2) A Bathymetric System

- Collection Module: Various independent field (ship-based) surveys of bathymetric 3d soundings over a very large area of the ocean, and crude estimates of their *a priori* accuracy; surveys follow ship tracks interspersed throughout the area.
- Value-Added Processing Module: Weighted combination/spline fit of the survey-data to generate a grid of estimated depth at specified horizontal locations over the entire area of interest, including corresponding predicted accuracy at each grid location.
- Exploitation Module: Generation of various nautical products, including predicted accuracy, to enhance navigation safety.

There are many possible NSG Geolocation Systems of interest, each with their own major modules, where each module requires its own relevant statistical error model(s) in order for the overall system to perform properly. Thus, this document does not address the accuracy or predicted accuracy (error models) of specific systems or modules. Instead, it provides over-all definitions and recommended standard practices applicable to all. This includes a “tool box” of applicable top-level statistical error models from which to select and populate. Other documents can then address specific systems and modules in an integrated and consistent fashion based on the information provided in this document.

Note: This document does present examples in some sections that are based on various aspects of image-based geopositioning for convenience and specificity; however, the same demonstrated principles apply across the entire scope of the NSG.

In summary, this section presented examples of major modules within NSG Geolocation Systems, consistent with the Figure 4.3-1 summary of a major module’s state S consisting of data, a state-vector describing the relevant state of the data, and a statistical error model corresponding to the state vector. The state vector is usually much smaller than the data itself. For example, the data may correspond to a set of images (pixels), and the state vector to the relevant metadata (time series of sensor position, attitude, etc.) for the images which enables extraction of geographic information. The statistical error model corresponds to the error in the state vector relative to truth, typically well-defined but unknown.

4.5 Geolocations and Coordinate Systems

In this document and underlying TGD 2 documents, both the state vector and its error are assumed to correspond to geolocations or values required to generate geolocations, such as sensor and sensor platform metadata. Therefore, for example, errors in the classification and attribution of features are not considered explicitly.

Geolocations are represented in various coordinate systems based on the World Geodetic System standard, WGS 84: Cartesian coordinates (x-y-z) and Geodetic coordinates (geodetic latitude, longitude, and height above the ellipsoid). Cartesian coordinate systems can either be Earth-Centered-Fixed (ECF) or local tangent plane, such as East-North-Up (ENU). Regardless the coordinate system used to represent geolocations, geolocation errors and corresponding statistics are recommended as represented in ENU. For a group of geolocations in a common and reasonably-sized area of interest, a common ENU coordinate system is recommended, i.e., one fixed origin near their “center” geolocation. A reasonably-sized area of interest is approximately no larger than a 1 degree x 1 degree cell (in latitude and longitude) over the earth's surface. On the other hand, if geolocations are to be considered on an individual basis, it is recommended that the origin of each geolocation's ENU coordinate system correspond to the geolocation's ECF coordinates. This will more precisely preserve the direction of "up" for each geolocation.

Deterministic and vetted transformations should be used to transform coordinates from one to another of the above coordinate systems. Transformation of errors and their statistics from one coordinate system to another is a form of error propagation and is based on corresponding first-order Taylor Series expansions (see TGD 2a). Finally, note that the WGS 84 reference is refined periodically; thus, it is important to time tag geolocation coordinates such that the corresponding WGS 84 reference can be determined at a later date if various coordinates are to be compared. See Section 5.13 of this document on the provenance of predicted accuracy for further details.

4.6 Accuracy versus Predicted Accuracy in the NSG: Examples

The title of this document starts with the term "Accuracy and Predicted Accuracy in the NSG". So, at the top-level and as relevant to the NSG, what is "accuracy" per se and how does it differ from "predicted accuracy"? Examples presented in this section support the earlier introductory discussion on accuracy and predicted accuracy. They also provide a "look-ahead" to many of the concepts discussed in Section 5.

4.6.1 Example focused on an overall Geolocation System

Let us assume an NSG Geolocation System that utilizes a commercial satellite-based imaging system, where exploitation consists of extracting the 3d location of a target of interest that is identified and measured in a pair of (stereo) images that were imaged on the same satellite pass and that cover approximately the same portion of the earth's surface. The imaging system is assumed to use the same specific sensor or collection of sensors of the same type. Many such commercial systems are operational today and utilized via industry partnerships and agreements throughout the NSG. Naturally, we are interested in the "accuracy" of such a geolocation system (Figure 4.6.1-1), and in particular, the accuracy of extracted geolocations. That is, the accuracy of an extracted 3d location of an arbitrary target from an arbitrary pair of images – ranging from thousands of past pairs to thousands of future pairs of images.

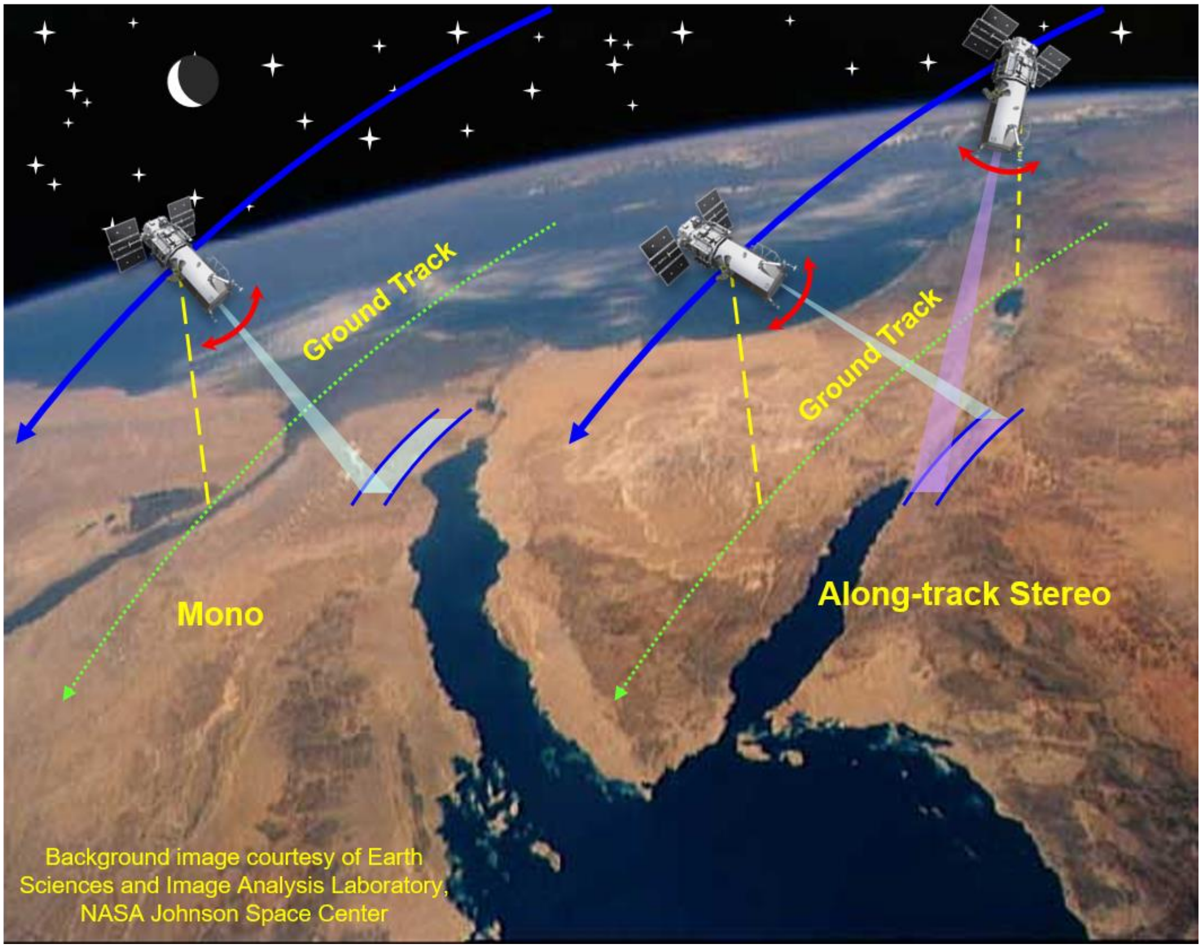


Figure 4.6.1-1: Example of a Stereo Electro-Optical (EO) Imaging System – right side of the above graphic

Accuracy

Accuracy is then defined as follows for such a system: for an arbitrary target location and an arbitrary pair of stereo images, it is 90% probable that horizontal (radial) extraction error is less than 5 meters and 90% probable that vertical extraction error is less than 6 meters, i.e., CE90$\leq$ 5 meters and LE90 $\leq$ 6 meters. Accuracy can correspond to a general characterization of the system or to actual system requirements; when the latter, CE90 and LE90 are sometimes referred to as specCE90 and specLE90, respectively.

CE90 and LE90, or alternatively CE and LE at different specified levels of probability, are used for the specification of system accuracy because they are practical: simple scalars, and easy to understand as detailed in Section 5.6. The actual values of CE90 and LE90 are typically determined by system design supplemented by the analysis of sample statistics, where the samples of error correspond to test sites containing ground truth or surveyed geolocations.

Note: The specific values of 5 and 6 meters for CE90 and LE90, respectively, in the previous paragraph are notional and for purposes of illustration. Also, the specification of accuracy requirements is necessarily a little more formal than as depicted in this section of the document – see Section 5.1.

Accuracy, as defined above, certainly provides us with a good overall picture as to what to expect in terms of geolocation errors for an arbitrary target extraction. In fact, it is essential for the NSG – but not enough. The "missing piece" of information is "predicted accuracy".

Predicted Accuracy

Predicted accuracy, as defined in these technical guidance documents, refers to an arbitrary but specific extraction of a geolocation. It does not refer to an arbitrary collection of past, current, or future geolocations as is applicable to the accuracy of the Geolocation System.

Predicted accuracy includes population of a corresponding detailed statistical error model, generated simultaneously with the target's 3d location via a MIG solution if an image-based sensor geolocation system. The MIG solution or "extraction" (subsection 5.9.2) is the output of a WLS estimator and takes advantage of the additional information that is available: (1) the specific imaging geometry of the stereo pair, as opposed to its possible operational range, (2) a specific prediction of the corresponding image metadata's expected magnitude and correlation of errors, provided in the metadata along with the specific sensor position and attitude values, and (3) a specific prediction of the expected magnitude and correlation of errors in the actual measurement of the target in the images, which is target feature/surrounding terrain characteristic-dependent. With this additional information, the MIG can provide an optimal solution of 3d location, including its error covariance matrix, a "custom-made" statistic-based description of the solution's error. Finally, regarding nomenclature, "predicted accuracy" refers to an extraction that has already occurred, not a future extraction.

The statistical error model for the extracted geolocation includes the $3x3$ error covariance matrix ($C_{\epsilon X}$) which specifies the expected magnitude and the correlations (inter-relationships) of the various components (x-y-z) making up the 3d location error. The error covariance matrix can also be used to compute and render an equivalent 90% probability error ellipsoid.

A 90% (probability) error ellipsoid corresponding to a typical but specific extraction is illustrated in Figure 4.6.1-2. The 90% error ellipsoid is centered at zero with a 90% probability that the solution 3d error resides within the ellipsoid. The predicted mean-value of error is assumed zero, as typically the case. A 90% confidence ellipsoid is identical except that it is centered at the solution location with a 90% confidence that the true target location resides within the ellipsoid

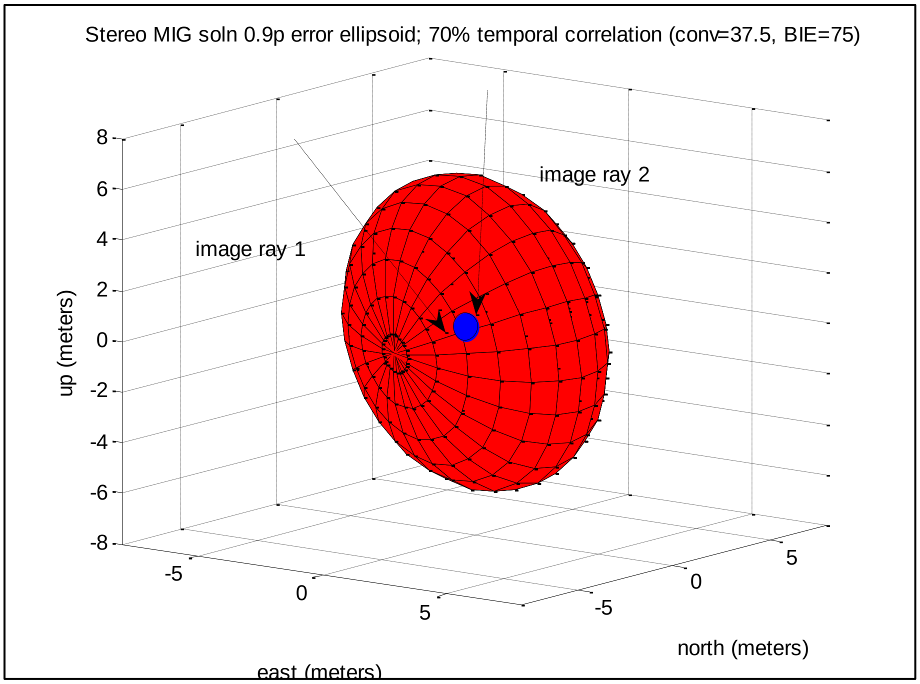


Figure 4.6.1-2: 90% probability error ellipsoid (image rays correspond to line-of-sights for electro-optical imaging system) corresponding to predicted accuracy of a specific geolocation solution; actual 3d error not shown but should reside within the ellipsoid with a probability of 90%

The error covariance can also be used to generate CE90 and LE90 (aka predCE90 and predLE90), which specify less information than the error covariance matrix or 90% probability error ellipsoid, but are convenient summaries and can be compared directly to the accuracy specification for the Geolocation System in general. The fact that CE90 and LE90 contain less information than the error covariance matrix is easily seen as follows: the $3x3$ error covariance matrix is symmetric and corresponds to 6 unique numbers, and CE90 and LE90 correspond to one unique number each.

Values of CE90 = 4 meters and LE90 = 5 meters correspond to the above specific solution, and are also illustrated in Figure 4.6.1-3 below. A CE90-LE90 error cylinder combines these two scalar accuracy metrics, is a convenient visual aid, and is illustrated in Figure 4.6.1-4.

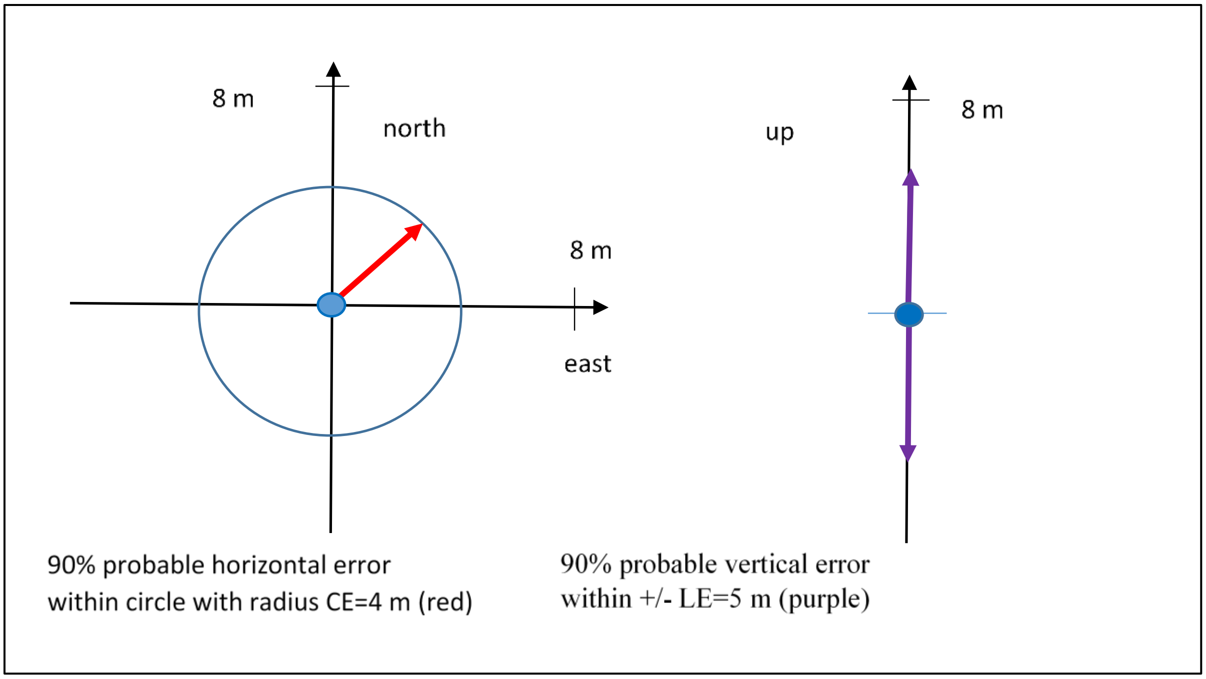


Figure 4.6.1-3: Corresponding 90% CE (CE90) and 90% LE (LE90) summaries contain less information for a specific solution than does the error covariance matrix itself or 90% probability error ellipsoid

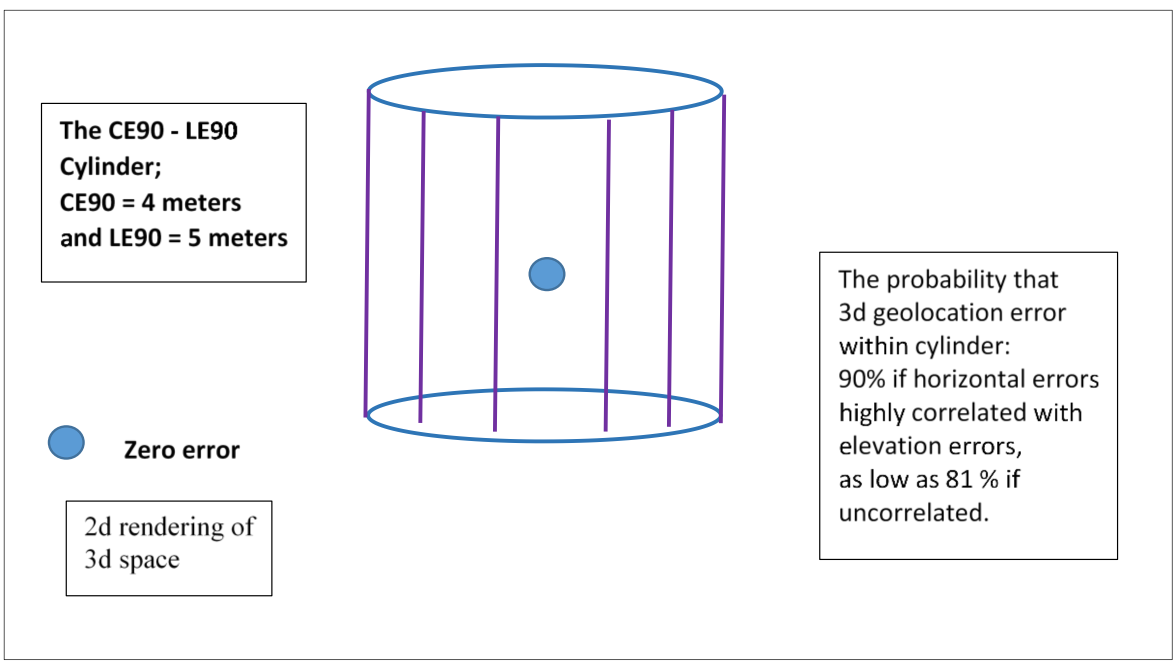


Figure 4.6.1-4: Corresponding CE90 – LE90 (error) Cylinder

CE90 and LE90 should only supplement the error covariance matrix, never replace it. Note that an error ellipsoid can be much more elongated than in Figure 4.6.1-2, such that a CE90 and LE90 representation alone would be even more problematic if they were to replace the error covariance matrix or 90% probability error ellipsoid.

See Section 5.5.1 for further details regarding the relationship of an error covariance matrix to probability ellipsoids or confidence ellipsoids, including their equivalence.

Predicted Accuracy caveat

If so caveated, predicted accuracy can also correspond to a hypothetical extraction of a specific geolocation, such as that in support of sensor tasking. The extraction makes use of specific, but hypothesized, sensor-to-geolocation geometry, and the same extraction algorithm and *a priori* error models as would be used for an actual (operational) extraction. No actual measurements are incorporated, and measurements are either simulated or not used at all. If the latter, only predicted accuracy is computed by the extraction algorithm, not the geolocation.

Predicted Accuracy benefits

The above discussion illustrates that predicted accuracy for a specific location, a MIG solution in this example, contains more detailed information about corresponding errors than does the top-level specification of accuracy for an arbitrary geolocation or extraction. This is due to both a more detailed description of errors via the error ellipsoid (error covariance matrix) than from the use of predCE90 and predLE90 alone, as well as from the fact that the predCE90 and predLE90 generated from this error covariance matrix (4 and 5 meters, respectively, for the above example) differ from the "generic" values of specCE90= 5 meters and specLE90=6 meters, respectively, used in the accuracy specification for an arbitrary extraction.

Furthermore, the predCE90 and predLE90 corresponding to the predicted accuracy of a specific extraction can differ from those specified for system accuracy in a much more dramatic way than for the above example. In particular, they could convey that there is a 90% probable 3 meter horizontal extraction error and a 90% probable 4 meter vertical extraction error if imaging geometry is in the "sweet spot", the estimates of the image metadata are good, and the target "stands out" in the imagery. More importantly, if imaging geometry is near the edge of its operational limit, the estimate of image metadata worse than usual, and the target "fuzzy" in the image due to weather conditions or ambiguity of definition, they could convey that there is a 90% probable 6 meter horizontal extraction error and a 90% probable 11 meter vertical extraction error – a critical piece of information for any actionable intelligence that is based on the extracted target location.

Also, as explained later in this document, the error covariance matrix ($C_{\epsilon X}$), the key ingredient in the statistical error model, allows for optimal use of the extracted location in "down-stream" value-added processing, such as fusion. Section 5.6.2.2 presents an example of fusion that yields an approximate 10x improvement in fusion accuracy for the combination of two different estimates of the 3d location of a common target of interest when the estimates' error covariances are used to combine (fuse) the estimates instead of just their corresponding CE and LE summaries.

In summary, the availability of predicted accuracy for each specific geolocation that is extracted is a critical piece of information. Furthermore, the reliability of predicted accuracy is also important and relies on realistic error models for all significant errors affecting the geolocation or estimator's solution.

4.6.1.1 Geolocation System variations: external elevations and sensor metadata

The first variation of the above Geolocation System that we are interested in corresponds to the left side of Figure 4.6.1-1 ("Mono") and is based on single-image extractions that rely on external estimates of elevation. It also more generally represents any Geolocation System with sensors whose 2d measurements need to be augmented with external elevation in order to extract 3d geolocations. This variation is discussed in Appendix B, including the form for its specified accuracy requirements, which are necessarily somewhat different than those for the stereo-image system detailed earlier.

The second variation of the above Geolocation System that we are interested in concerns estimates of objects other than geolocations per se, and their corresponding accuracy and predicted accuracy requirements. For example, estimates for improved sensor metadata (e.g., sensor pose), instead of, or in addition to, estimates for geolocations per se. This variation is discussed in Appendix C.

4.6.2 Example focused on an arbitrary but specific geolocation

This section presents an example that is focused on the predicted accuracy of an arbitrary but specific geolocation associated with a Geolocation System at a level "deeper" than that which was provided in Section 4.6.1 for a commercial satellite-based imaging system. It also further illustrates the relationship between predicted accuracy and the accuracy of the Geolocation System. As discussed in Section 4.6.1, the accuracy of a Geolocation System is applicable to an arbitrary collection of past, current, and/or future geolocations extracted by the system or its "down-stream" users. Predicted accuracy of a Geolocation System is applicable to each arbitrary but specific geolocation extracted by the system at the time at which it is extracted, i.e., generated as part of the extraction itself. The predicted accuracy generally varies across specific geolocations.

The Geolocation System and its sensors are not specific in this example and could correspond to virtually any NSG Geolocation System and corresponding sensors. The geolocation is assumed to correspond to the extraction of an arbitrary but specific feature's geolocation using a (near) optimal estimator, such as a WLS estimator, which estimates the geolocation using sensor-based measurements related to the geolocation, computes a corresponding solution error covariance matrix, and typically resides within the Exploitation Module of Figure 4.1-1. The estimate corresponds to lowest expected magnitude of solution error or estimator "cost". In this example, error corresponds to the error in the estimator's solution X for the feature's 3d geolocation, and is considered a 3d random vector because the measurements used by the estimator contain random errors and are propagated into solution errors by the solution process, a form of rigorous error propagation.

Figure 4.6.2-1 illustrates the top-level concepts and interrelated roles of estimator, solution error, and predicted accuracy for a specific geolocation. The 90% confidence ellipse is generated from the upper left $2x2$ portion of the $3x3$ error covariance matrix, the latter applicable to 3d geolocation error. A 90% confidence ellipsoid is similar, but applicable to 3d geolocation error and based on the full $3x3$ error covariance matrix. Both the $2x2$ and the $3x3$ error covariance matrices are relative to the local tangent plane coordinate system. In general, the predicted accuracy, or error covariance matrix and corresponding confidence ellipse or ellipsoid, will differ somewhat for each geolocation solved for.

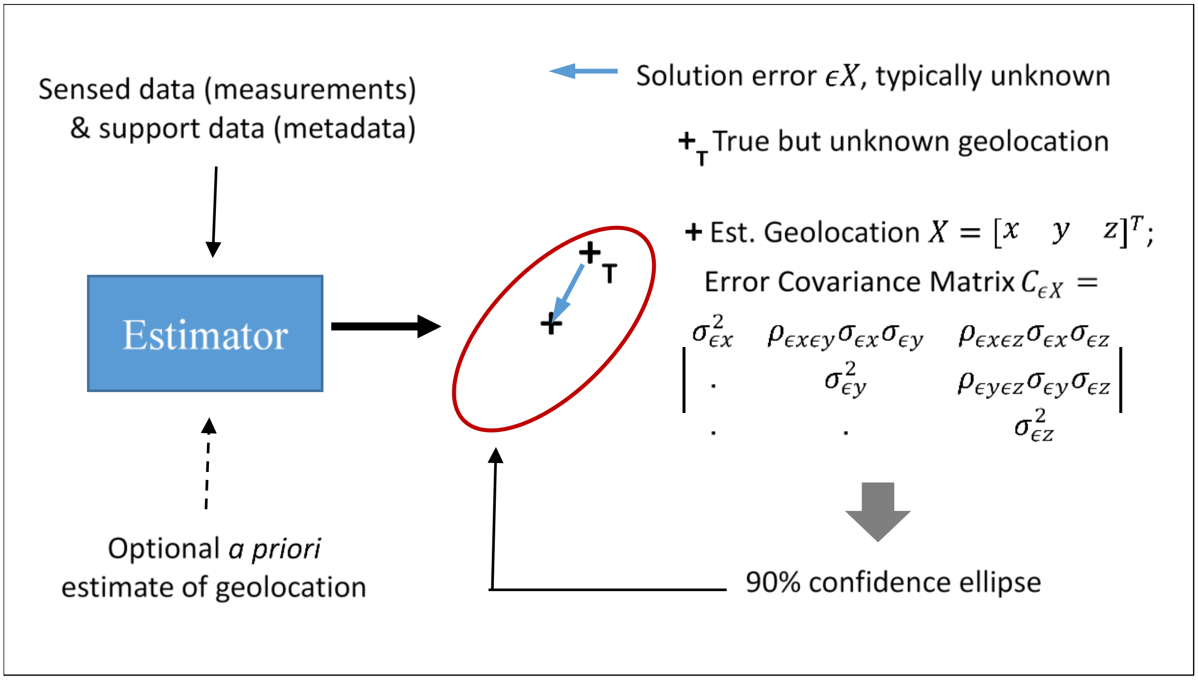


Figure 4.6.2-1: An overview of the relationships between Estimator, Solution Error, and Predicted Accuracy for an arbitrary but specific geolocation; Predicted Accuracy corresponds to the error covariance matrix $C_{\epsilon X}$ from which the 90% confidence ellipse is derived

The particular representation for the error covariance matrix $C_{\epsilon X}$ in the figure is based on the use of standard deviations of error and correlation coefficients; for example, $\sigma_{\epsilon x}$ is the standard deviation of the x-component error ϵx and $\rho_{\epsilon x \epsilon y}$ is the correlation (coefficient) between the x-component and the y-component errors ϵx and ϵy, etc. The correlation coefficient is a measure of the expected similarity of errors, and when equal to zero, represents uncorrelated errors, or more generally, independent errors, when errors are also assumed Gaussian distributed. As discussed later, $C_{\epsilon X}$ actually represents the expected dispersion of errors about the mean-value of error, the latter assumed equal to zero in this example as almost always the case for predictive statistics.

<u>An accurate geolocation with reliable predicted accuracy</u>

The specific solution illustrated above should correspond to an accurate geolocation with reliable predicted accuracy, which means that the specific solution has the following two properties:

1) The geolocation meets or exceeds (smaller accuracy values) the accuracy requirements for the Geolocation System for an arbitrary geolocation, i.e., the geolocation is an **accurate geolocation.**

2) The solution error is consistent with the solution's predicted accuracy or statistical description, i.e., the predicted accuracy is a **reliable predicted accuracy**.

These two properties are illustrated graphically in Figure 4.6.2-2 below. The predicted 90% error ellipse is equivalent to the error covariance matrix contained in the predicted accuracy's statistical description. Also, multiple independent realizations of the specific solution for the same geolocation were performed, corresponding to the same basic measurements, but with different realizations of measurement error. This changes the solution (error) each time but not its predicted accuracy. Note that only one realization was illustrated with the corresponding 90% confidence ellipse in Figure 4.6.2-1.

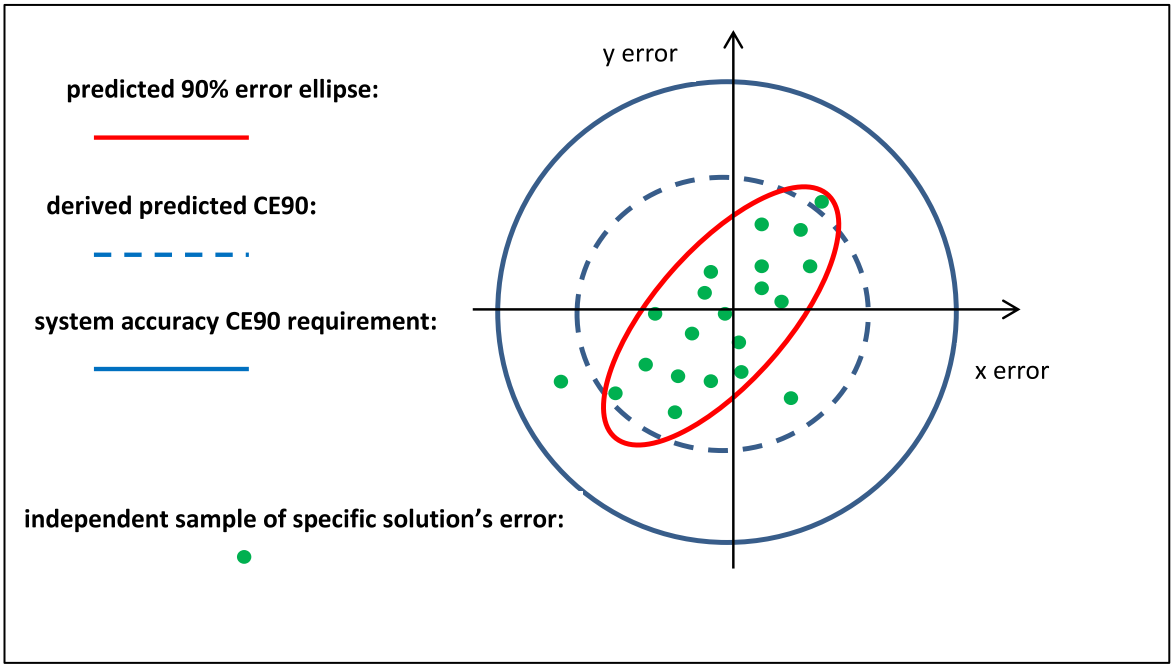


Figure 4.6.2-2: Independent realizations of a specific solution and corresponding errors; the solution corresponds to an accurate geolocation with reliable predicted accuracy

Approximately 90% of the independent samples of solution error are within the 90% error ellipse in Figure 4.6.2-2, which corresponds to reliable predicted accuracy. In addition, at least 90% of the independent samples of solution error are within the system CE90 requirement, the radius of the outer (blue) circle in the figure, which corresponds to an accurate geolocation.

The derived predicted CE90 (dashed blue circle), also presented in Figure 4.6.2-2, is computed from the error covariance matrix and allows for convenient comparison to the system CE90 requirement. In addition, the system CE90 requirement is sometimes termed "specCE90" and the derived predicted CE90 is sometimes termed "predCE90".

In general, characteristic #1 (accurate geolocation) does not necessarily imply characteristic #2 (reliable predicted accuracy) and vice versa, although both are satisfied in Figure 4.6.2-2, as desired. Figure 4.6.2-

3 illustrates various instances of the four possible combinations, with the derived predicted CE90 circle left out to keep things from getting too cluttered:

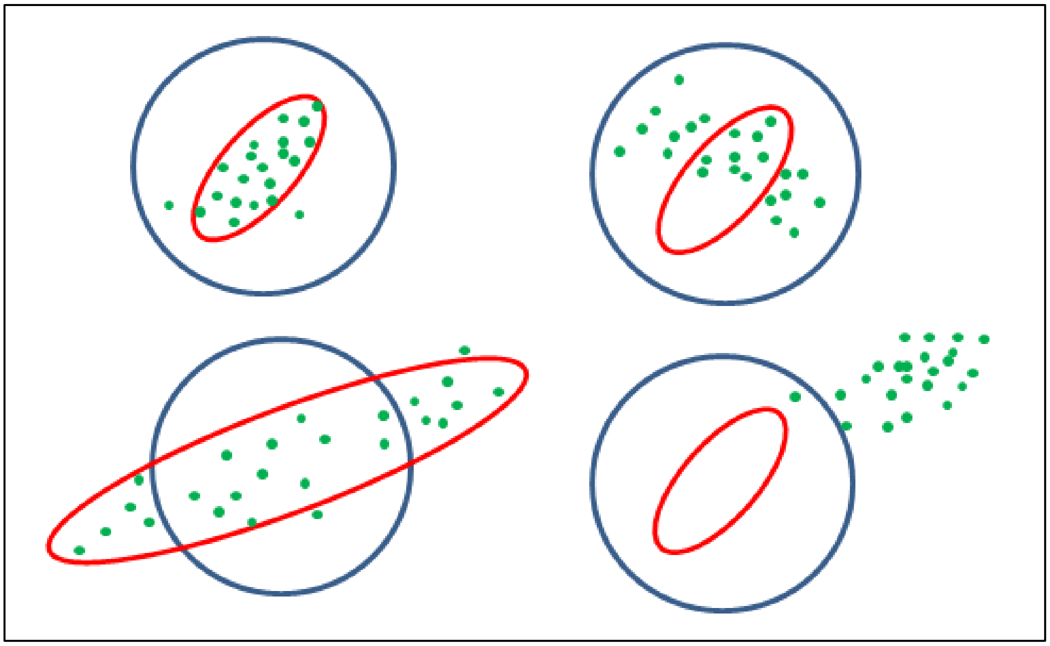


Figure 4.6.2-3: accurate geolocation and reliable predicted accuracy (upper left), accurate geolocation and unreliable predicted accuracy (upper right), inaccurate geolocation and reliable predicted accuracy (lower left), and inaccurate geolocation and unreliable predicted accuracy (lower right)

How do we know if a geolocation is accurate with reliable predicted accuracy?

Operationally, there is of course only one realization of a specific solution, and its corresponding geolocation error is unknown. So how are we reasonably sure that the specific solution corresponds to an accurate geolocation with corresponding reliable predicted accuracy? We rely on two factors:

1) The Geolocation System having specified accuracy requirements that were validated and predicted accuracy requirements that were validated – see Section 5.1 of this document for a corresponding overview and TGD 2c (Specification and Validation) for details. Validation is based on the use of multiple independent samples of geolocation error and corresponding predicted accuracies (error covariance matrices) over multiple locations.
    a) This addresses arbitrary geolocations.
2) The Quality Control (QC) of the specific solution of interest and performed by the estimator – see Sections 5.9.3 and 5.9.4 of this document for a corresponding overview and TGD 2d (Estimators and their QC) for details.
    a) This addresses the specific solution.

4.7 Guide to Technical Content

Now that a general overview of an NSG Geolocation System and its major modules have been presented, including relevant definitions for accuracy and predicted accuracy, a guide to further technical content is presented prior to Section 5. In particular, an overview of the various level 2 Technical Guidance Documents and their interrelationships is presented in Section 4.7.1, followed by a detailed technical guide to the content of Section 5 of this document that is presented in Section 4.7.2.

4.7.1 Overview of the level 2 Technical Guidance Documents

Figure 4.7.1-1 presents an overview of the level 2 Technical Guidance documents and their inter-relationships:

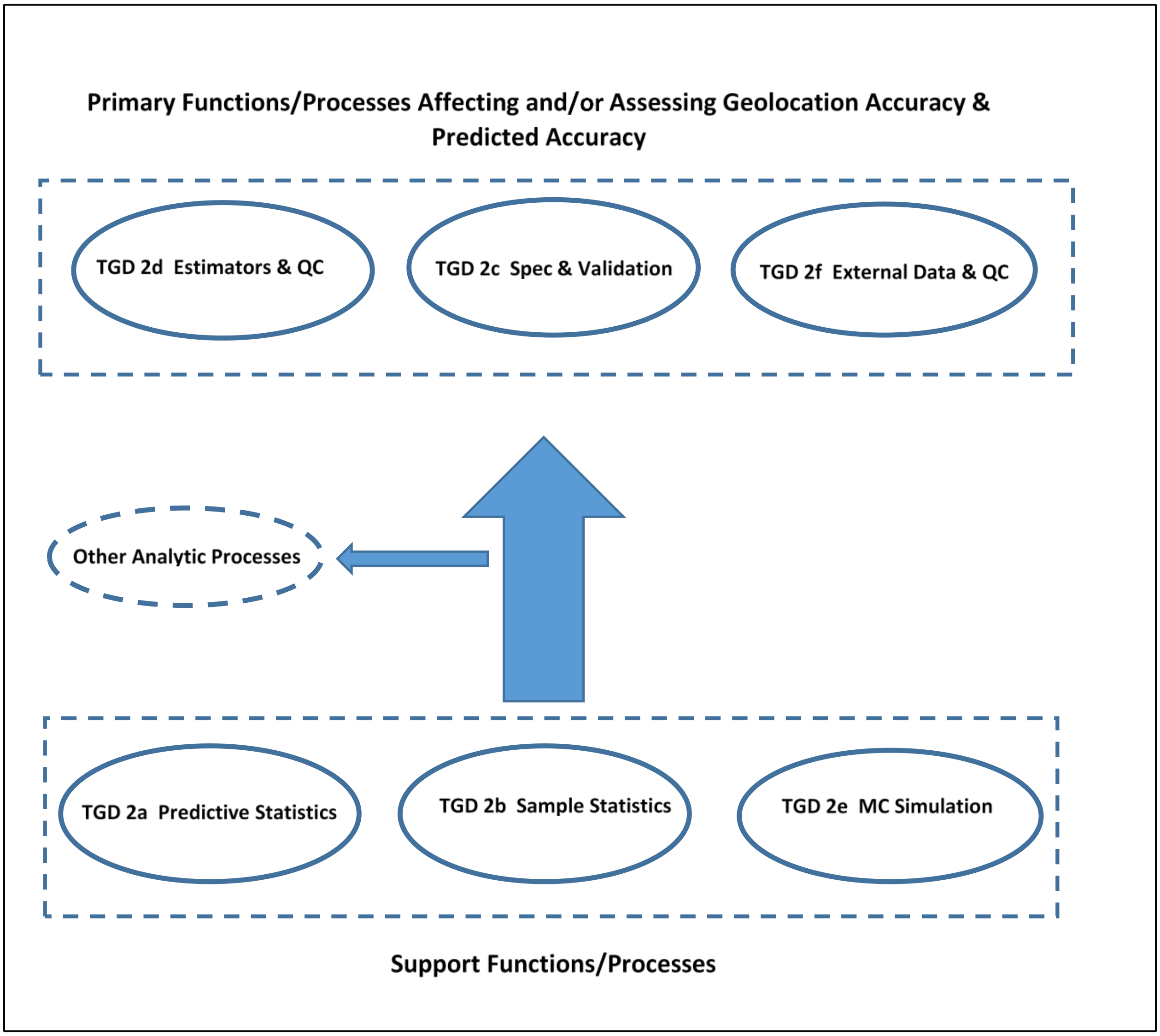


Figure 4.7.1-1: The roles played by the various level 2 Accuracy and Predicted Accuracy Technical Guidance Documents in support of an NSG Geolocation System

The upper level of the figure contains three documents which provide detailed technical guidance for the generation of accurate geolocations with reliable predicted accuracies.

The first document corresponds to TGD 2d which presents recommendations for the development and implementation of estimators in an NSG Geolocation System, including the Quality Control (QC) of their outputs to ensure (near) optimal estimates and corresponding reliable error covariance matrices or predicted accuracy. The second document corresponds to TGD 2c which presents recommendations for the specification, validation, and overall assessments of accuracy and predicted accuracy of an NSG Geolocation System. The third document corresponds to TGD 2f which presents recommendations for the Quality Assessment of External Data used within the NSG, such as crowd-sourcing data, which typically contains no corresponding pedigree or predicted accuracy, and commodities data, which typically contains little corresponding pedigree. Processing includes the generation of populated predicted accuracy models, when feasible.

These three documents correspond to the "primary" NSG Geolocations functions associated with (1) the NSG-internal generation of accurate geolocations and related sensor metadata, including their reliable predicted accuracies, (2) their NSG-internal specification and validation of corresponding accuracy and predicted accuracy requirements, and (3) the NSG-internal assessment of the quality, reliability, and accuracy of geolocation related data generated external to the NSG but used internally.

The lower level of Figure 4.7.1-1 corresponds to NSG "support" functions and contains three documents which support the above primary functions. The first document corresponds to TGD 2a which presents recommendations for the appropriate generation and use of predictive statistics. The second document corresponds to TGD 2b which presents recommendations for the appropriate generation and use of sample statistics. The third document corresponds to TGD 2e which presents recommendations for the appropriate use of Monte Carlo simulation in the support of error modeling.

The following Tables 4.7.1-1 through 4.7.1-3 present overviews of the contents of all six documents: TGD 2a through TGD 2f, in that order.

Table 4.7.1-1: Overview of the Technical Guidance Documents regarding Statistics – TGD 2a: Predictive Statistics, and TGD 2b: Sample Statistics

TGD document:	**TGD 2a: Predictive Statistics**		**TGD 2b: Sample Statistics**
Elements:	Mean value, covariance matrix, correlation function, probability density function, cumulative probability distribution, various related statistics and metrics.		Sample counterparts to elements of predictive statistics.
Applications:	*A priori* error modelling: random variables, random vectors, stochastic processes, random fields. Rigorous error propagation. Representation of predicted accuracy. Support of operational decisions.		Assessing accuracy and predicted accuracy performance: sample statiscs of error samples. Validation of accuracy and predicted accuracy requirements: comparison of sample statistics to specs. Tuning of *a prior* i error models.
Details and Methods:	Covariance matrices: generation, representation, dissemination. Confidence ellipses and ellipsoids: how to compute/render. Comparison of covariance matrices: A<B and A<=B, meaning and applications. Scalar Acccuracy Metrics: CE90 / LE90 / SE90 and other probability levels, how to compute/render, pros and cons of use. Directed percentiles: prob of error in specified direction. Method of Covariance Intersection: proper estimate X in the presence of unknown correlation of errors.		Order statistics. Classical statistics. Confidence intervals. Computation of sample CE90 / LE90 / SE 90, etc. Hypothesis tests.
TGD1 referrals:	Sections 5.4 - 5.8		Section 5.4

Table 4.7.1-2: Overview of the Technical Guidance Documents regarding Processes – TGD 2c: Specification and Validation, and TGD 2d: Estimators and their Quality Assurance and Quality Control

TGD document:	**TGD 2c: Specification and Validation**		**TGD 2d: Estimators and their QA/QC**
Descriptions and methods for:	Specification of Accuracy Reqts. Validation of Accuracy Reqts. Assessment of Accuracy perf. Specification of Pred Acc Reqts. Validation of Pred Acc Reqts. Assessment of Pred Accuracy perf. Requirements correspond a Geolocation System and its "extracted" 3d geolocations. Similar methods for Relative Acc and Pred Relative Acc. Type I and Type II validation errors.		Estimator characteristics: batch vs. sequential, cost function, optimality, ... Estimator implementations: Weighted Least Squares, Kalman filter,... Estimator effects on Accuracy and Predicted Accuaracy: 3d geolocations and/or improved sensor metadata, ... The difference between Quality Assurance (QA) and Quality Control (QC) of estimators and QA/QC's ensurance of: (near) optimal solutions and reliable predicted accuracies.
Related concepts and details:	Levels of confidence in assessments. Recommended # independent samples of geolocation error via "ground truth" . Specifiable levels of pred accuracy fidelity. The need for relevant and verifiable speciffications (reqmts) for both accuracy and pred accuracy.		Estimator QA/QC based on: editing of measurements, reference variance , confidence intervals, solution conv. detection, plots/trend analyses, ... QA/QC use of internal data (msmnt residuals) and occasional grnd truth. Correlated and uncorrelated msmnt residuals and their mean-value and covariance matrix.
TGD1 referrals:	Section 5.1		Sections 5.8 amd 5.9

Table 4.7.1-3: Overview of the Technical Guidance Documents regarding Processes – TGD 2e: Monte Carlo Simulation, and TGD 2e: External Data and its Quality Assurance

TGD document:	**TGD 2e: Monte Carlo Simulation**		**TGD 2f: External data and its QA**
Descriptions and/or methods for:	Assessment of effects of various error sources on Accuracy and Pred Accuracy based on Monte Carlo simulation: independent samples generated based on specifiable predictive statistics, samples analyzed based on sample statistics, effects of errors on nonlinear and/or complicated systems. Simulated errors can correspond to: random variables, random vectors, stochastic processes, and random fields.		NSG use of External data: outsourcing; crowd sourcing, commoditites. Examples of External data: Small-Sat imagery, 3d Point Clouds, Crowd-sourced maps. The QA/QC of External data within the NSG: assessing it reliability and accuracy, categorized by source or vendor, date-range, etc. Populatation of acc assessment and pred acc models for NSG use.
Related concepts and details:	Gaussian distribution of errors usually modelled but technique also detailed for arbitrary distribution. Convenient technique for simulation of random vectors consistent with specifiable mean-value and covariance matrix. Technique for the fast generation of simulated realizations of stochastic processes or random fields over 1D to 4D (x ,y, z, time) grids: specifiable correlation of errors. Enables study of the effects of errors over a broad array of applications where strictly analytic methods are not viable; can also be embeded in various product generation tasks.		The growing importance of external data within the NSG. Difficulties associated with its assessment: lack of pedigree, few error samples, … Techniques based on comparison of independent sources. Techniques based on sample stats tailored to few error samples. Importance of management and dissemination of assessments within the NSG and corresponding recommended methodologies. Recommended future R&D.
TGD1 referrals:	Section 5.11		Section 5.12

4.7.2 Detailed Guide to Section 5 of this level 1 Technical Guidance Document

The following outlines the contents of Section 5 of this document, Methods, Practices, and Applications for Accuracy and Predicted Accuracy:

- Section 5.1: Performance Specification and Validation – describes the methodology for the verification, validation, and overall assessment of an NSG Geolocation System's accuracy and its predicted accuracy capabilities; presents an example of the specification and validation of accuracy and an example of the specification and validation of predicted accuracy
- Section 5.2: Statistical Error Model Overview – provides an overview of the contents of the statistical error model associated with the predicted accuracy of an NSG module's state vector(s)
- Section 5.3: Types of representation of Error: Random Vector, Stochastic Process, Random Field – describes the various types of representation of errors modeled in the statistical error model
- Section 5.4: Statistical Categories: Predictive and Sample – provides an overview of the statistical error model's predictive statistics versus sample-based statistics
- Section 5.5: Error Covariance Matrix – describes the key statistic in the statistical error model, the error covariance matrix; how to generate corresponding probability-based error or confidence ellipsoids; examples of the importance of generating and using the full error covariance matrix; applications involving comparisons between error covariance matrices and the Method of Covariance Intersection
- Section 5.6: Scalar Accuracy Metrics: Linear Error (LEXX), Circular Error (CEXX), and Spherical Error (SEXX) at specified probability level XX% – how to generate the ubiquitous and probability-based scalar accuracy metrics (percentiles) from the error covariance matrix; their desirable features as well as their limitations
- Section 5.7: Representation/Dissemination of Error Covariance Matrices – an overview of the recommended techniques for both the representation and the dissemination of error covariance matrices associated with errors represented as random vectors, stochastic processes, and random fields; particularly useful for very large error covariance matrices associated with multi-state vectors
- Section 5.8: Rigorous Error Propagation – its definition and overview of its importance, particularly as associated with estimators, e.g., Weighted Least Squares (WLS)
- Section 5.9: Estimators: General Overview – an overview of estimators in the NSG and their important characteristics; details of the MIG (WLS) estimator; corresponding standard practices for estimator optimality and Quality Assurance (QA) and Quality Control (QC)
- Section 5.10: Accuracy and Statistical Error Model Periodic Calibration – the recommended standard practice of the periodic calibration of the error models associated with an NSG system's accuracy and predicted accuracy
- Section 5.11: Monte-Carlo Simulation of Errors for Complex Systems – the importance of Monte-Carlo simulation of errors associated with accuracy and predicted accuracy, particularly corresponding to "black-box" systems, as well as applications involving large amounts of data or non-linear equations; simple examples of the generation of independent realizations of random vectors, both Gaussian distributed as well as arbitrarily distributed, as well as more general

examples of Monte-Carlo simulation of random fields embedded in a bathymetric application and a non-linear MIG application

- Section 5.12: External Data and its Quality Assessment – overview of potential techniques and research for the difficult problem associated with the Quality Assessment, including quantifying accuracy and predicted accuracy, of NSG-external data (e.g., crowd-sourcing and commodities data); examples of representative commodities data and crowd-sourcing data; example of a recommended predicted accuracy model for commodities data
- Section 5.13: Provenance for Predicted Accuracy –the provenance of predicted accuracy, such as inclusion of the time-of-applicability as a standard practice; identification of applicable coordinate systems and datum, research for the automatic “adjustment” of historical predicted accuracy
- Section 5.14: Computer System Capabilities – the recommended use of available increased computer power associated with accuracy and predicted accuracy processing
- Section 5.15: Recommended Practices Overview – an overview of the recommended practices associated with accuracy and predicted accuracy in the NSG and a brief summary of the contents of this document.

In addition, Appendices B and C present important variations of the Geolocation System and its corresponding accuracy and predicted accuracy requirements that were described in Section 4.6.1. These variations concern Geolocation Systems that require external elevations (Appendix B), and requirements corresponding to objects other than geolocations, such as sensor metadata (Appendix C).

Appendix D presents miscellaneous but important supporting comments regarding Section 5.1 on Performance Specification and Validation.

5 Methods, Practices and Applications for Accuracy and Predicted Accuracy

5.1 Performance Specification and Validation

In addition to normal operations, accuracy and predicted accuracy also play a critical role in performance specification and validation of an NSG Geolocation System as outlined in Figure 5.1-1.

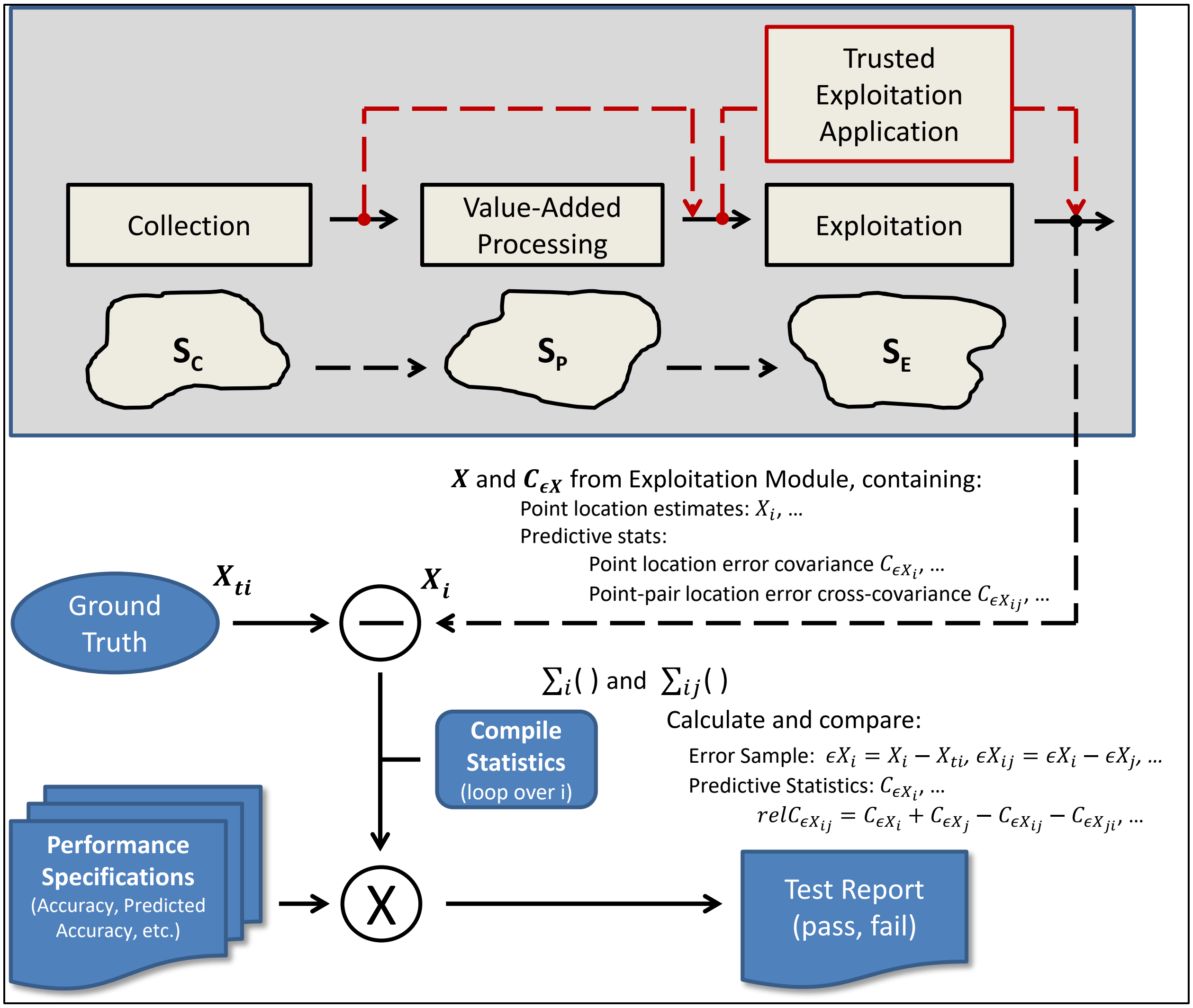


Figure 5.1-1: Validation of Accuracy and Predicted Accuracy Performance Specifications

The representative state vector X in the above figure and its error covariance matrix $C_{\epsilon X}$ are contained in the Exploitation module's general state S_E (Figure 4.1-1). The representative state vector X contains independent estimates of multiple geographic locations X_i, and its error covariance $C_{\epsilon X}$ is a block diagonal matrix that contains multiple error covariance matrices $C_{\epsilon X_i}$ (predicted accuracy) down its main diagonal corresponding to the errors in the X_i. More specifically, and assuming m independent estimates:

$$X = \begin{bmatrix} X_1 \\ X_2 \\ \cdots \\ X_m \end{bmatrix} \text{ and } C_{\epsilon X} = \begin{bmatrix} C_{\epsilon X_1} & 0 & 0 & 0 \\ 0 & C_{\epsilon X_2} & 0 & 0 \\ 0 & 0 & \cdots & \cdots \\ 0 & 0 & \cdots & C_{\epsilon X_m} \end{bmatrix},$$

where both $C_{\epsilon X}$ and the $C_{\epsilon X_i}$ are predictive statistics.

Error samples ϵX_i correspond to the difference between the estimated geographic locations X_i and corresponding ground truth X_{ti}. Sample statistics taken over the error samples are computed and compared to specified accuracy requirements. Sample statistics taken over error samples, normalized by their predicted accuracies ($C_{\epsilon X_i}$), are also computed and compared to specified predicted accuracy requirements. The latter requirements essentially state (quantitatively) that predicted accuracy shall reliably reflect the actual errors corresponding to a specific but arbitrary extraction.

In general, error samples are to be independent, requiring the simultaneous analysis of the multiple individual estimated state vectors X_i (e.g., 3d geolocations) and their individual error covariance matrices $C_{\epsilon X_i}$, each generated using an independent set of underlying data.

Both accuracy requirements and predicted accuracy requirements are recommended for a Geolocation System. The specification and validation of accuracy requirements are independent of the specification and validation of predicted accuracy requirements, other than the possible sharing of error samples.

For those readers interested in further details, Sections 5.1.1 and 5.1.2 go on to present relatively simple but quantitative examples of the specification and validation of accuracy and the specification and validation of predicted accuracy, respectively. Only absolute accuracy and predicted absolute accuracy are addressed in these sections, with TGD 2c providing further details. The specification and validation of relative accuracy and predicted relative accuracy are also detailed in TGD 2c, but are not summarized below for simplicity. Section 5.1.2 also relies on the use of error covariance matrices and corresponding error ellipsoids, both introduced earlier in Section 4 of this document, discussed further in some of the remaining sections of this document, and detailed in TGD 2a (Predictive Statistics) and TGD 2d (Estimators and their QC).

Note: the representative state vector X and its error covariance matrix $C_{\epsilon X}$ in Figure 5.1-1 are conceptual for ease of illustration; only the locations X_i and their error covariance matrices $C_{\epsilon X_i}$ are actually output from the Exploitation Module.

5.1.1 Specification and Validation of Accuracy

A typical specification for the accuracy of 3d geolocation involves a separate specification for horizontal accuracy and for vertical accuracy. We only address horizontal accuracy in this section of the document for simplicity – the underlying concepts for horizontal accuracy, vertical accuracy, and even 3d accuracy per se are virtually identical. We also assume that accuracy specifications correspond to $XX = 90\%$ probability levels, as is typical. Hypothetical values for the specified requirements are also used, as actual values are Geolocation System dependent.

Horizontal errors addressed below are 2d random vectors, $\epsilon X = [\epsilon x \quad \epsilon y]^T$, and their magnitude a scalar termed horizontal radial error, designated $\epsilon h = \sqrt{\epsilon x^2 + \epsilon y^2}$.

<u>Specification (requirement):</u>

The recommended form for specified accuracy requirement is as follows:

$\epsilon h_{XX} \leq CEXX_{spec}$,

where XX corresponds to a desired probability level, typically 90%.

Therefore, for specificity of example, and assuming $XX = 90$ % and a hypothetical value of $CE90_{spec} =$ 5.5 meters, we have the following accuracy specification:

$\epsilon h_{90} \leq CE90_{spec} = 5.5$ meters.

The specification requires that there is at least a 90% probability that an arbitrary geolocation's horizontal radial error is less than or equal to 5.5 meters.

More specifically, and with regard to the above specification, ϵh_{90} is defined as the 90th percentile of horizontal radial error $h = \sqrt{\epsilon x^2 + \epsilon y^2}$, i.e., $prob\{\epsilon h \leq \epsilon h_{90}\} = 0.90$. $CE90_{spec}$ is its specified upper bound. The actual definition of $CE90$ is identical to that of ϵh_{90}, used for legacy purposes, and is further detailed in Section 5.6. The above specification can also be written in the equivalent form:

$prob\{\epsilon h \leq CE90_{spec}\} \geq 0.90$, where ϵh is an arbitrary horizontal radial error.

Validation

Validation that the above specification is met is based on the use of order statistics of independent samples of horizontal (radial) error. The process requires no assumption regarding the probability distribution of errors nor their mean-value – a desirable and robust feature. The validation process computes both a best estimate of ϵh_{90} as well as a least upper bound for ϵh_{90} at a specified confidence level YY. The specified confidence level is associated with the validation process itself, and is not necessarily equal to the probability level XX associated with the accuracy specification.

The least upper bound

The least upper bound $lub\_\epsilon h_{90}$ is computed as a function of both the confidence level YY as well as the number of error samples n that are available, the latter used in order to account for the statistical significance of results associated with a finite number of error samples. The use of a least-upper-bound (one-sided confidence interval) is critical for the validation of accuracy, as the NSG must have confidence in its results.

Thus, carrying forward our example of the accuracy specification, where $XX = 90\%$ (or 0.90), and taking the confidence level YY into consideration, $lub\_\epsilon h_{90}$ is defined as satisfying the following condition:

$prob\{\epsilon h_{90} < lub\_\epsilon h_{90}\} \geq 0.YY$.

Or more specifically, for our example and for the use of a confidence level YY that is also equal to 90%, as is typical:

$prob\{\epsilon h_{90} < lub\_\epsilon h_{90}\} \geq 0.90$.

The validation process

The validation process computes $lub\_\epsilon h_{90}$ that satisfies the above inequality by the use of order statistics over n independent samples of horizontal radial error.

Correspondingly, if the computed $lub\_\epsilon h_{90} \leq CE90_{spec} = 5.5$ meters, validation is successful.

That is, validation passes: we are at least 90% confident that the accuracy requirement is met.

<u>A specific example</u>

The validation test is simple, straight-forward, and is also "plot-friendly" for additional insight and confidence in the validation results.

This is illustrated in Figure 5.1.1-1 corresponding to $n = 100$ independent and approximately identically distributed error samples that were simulated for this example consistent with a (true) $CE90 \cong 4.6$ meters. The blue circles correspond to the samples ϵh_k of horizontal radial error, the magenta line the value $lub\_\epsilon h_{90}$ computed from these samples, and the dashed red-line the best estimate of ϵh_{90} computed from these samples as well for ancillary information.

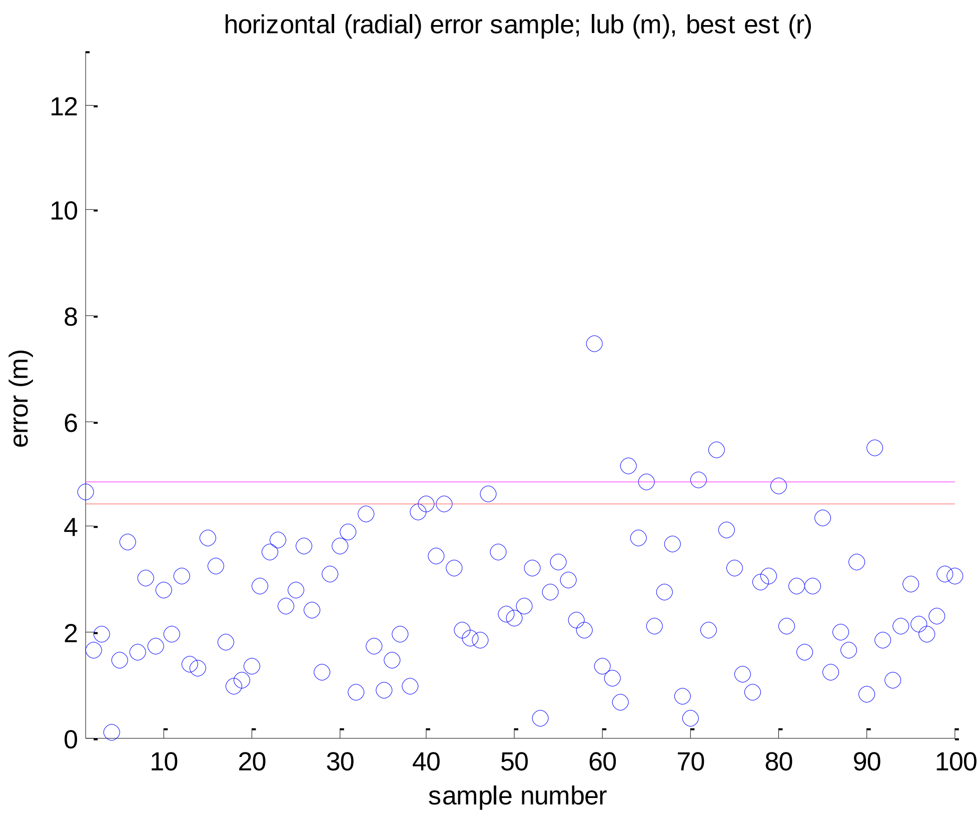


Figure 5.1.1-1: Example of the successful Validation of Horizontal Accuracy based on 100 i.i.d. samples of horizontal radial error; solid magenta line corresponds to $lub\_\epsilon h_{90} = 4.9$ m, the 95th ordered sample; dashed red line corresponds to the best estimate of $\epsilon h_{90} = 4.4$ m, the 90th ordered sample.

The best estimate of ϵh_{90} corresponds to the value of the 90th ordered sample and the least-upper-bound $lub\_\epsilon h_{90}$ corresponds to the value of the 95th ordered sample (samples ordered by ascending magnitude, not as shown in Figure 5.1.1-1 where they simply correspond to sample number on the x-axis). Note that selection of the 90th ordered sample for the best estimate of ϵh_{90} corresponds to (0.90 percentile x 100 order samples) = 90. The least-upper-bound $lub\_\epsilon h_{90}$ contains an additional "pad" (higher ordered sample number) that is required in order to ensure, at a YY =90% confidence, that the true but unknown ϵh_{90} is less than or equal to the computed least-upper-bound.

Validation was successful for this particular example:

$lub\_\epsilon h_{90} = 4.9 \leq CE90_{spec} = 5.5$; validation passes.

That is, we are at least 90% confident that the true and unknown value of the 90th percentile ϵh_{90} is less than the computed value $lub\_\epsilon h_{90}$ which is less than the specified requirement.

Note: the method used to determine the ordered sample number corresponding to $lub\_\epsilon h_{90}$ (e.g. 95th ordered sample for $n = 100$) is detailed in TGD 2c and essentially corresponds to the use of look-up tables and is automatically computed in provided pseudo-code. TGD 2c also refers to the appropriate sections in TGD 2b (Sample Statistics) for underlying theory.

5.1.2 Specification and Validation of Predicted Accuracy

The specification and validation of predicted accuracy requirements are necessarily somewhat more complicated than for accuracy requirements. They are designed to ensure that predicted accuracies, essentially the individual error covariance matrices associated with each of the individual geolocations extracted using the Geolocation System, reasonably and reliably reflect the statistical characteristics of corresponding errors – not only their expected magnitude, but their distribution assuming an approximate Gaussian probability distribution. The use of an assumed probability distribution for errors is necessary in order to ensure that that predicted accuracy is reliable in all of its applications, including the computation of error ellipsoids at different probability or confidence levels. Furthermore, when we say that errors are assumed to be Gaussian distributed, we are referring explicitly to errors (random vectors), not to their magnitudes, i.e., radial errors.

The following outlines the methods for the specification and the corresponding validation of predicted accuracy requirements for general insight. Again, only horizontal errors are assumed for simplicity. See TGD 2c for a more detailed overview, including specific quantitative examples, as well as algorithmic details for formal specification and validation. TGD 2c also addresses the applicability of these methods for the relatively infrequent case when errors are not, at least approximately, Gaussian distributed.

Specification and validation of predicted accuracy requirements address normalized errors – geolocation errors essentially normalized by their corresponding error covariance matrices that are provided along with the geolocations by the extraction process, e.g., by estimators. More specifically, errors are normalized corresponding to three different probability levels (99, 90, and 50%) in a manner equivalent to comparing each sample of horizontal (radial) error to the corresponding predicted radials applicable to each of three error ellipses (99, 90, and 50%).

One of the three predicted radials for a specific sample of horizontal error is the predicted 90% radial associated with the 90% error ellipse, and is illustrated in Figure 5.1.2-1. The predicted 90% radial is the magnitude of the radial vector (blue vector) of the 90% error ellipse and the horizontal radial error is the magnitude of the horizontal error (red arrow). The direction of the radial vector aligns with the direction of the horizontal error.

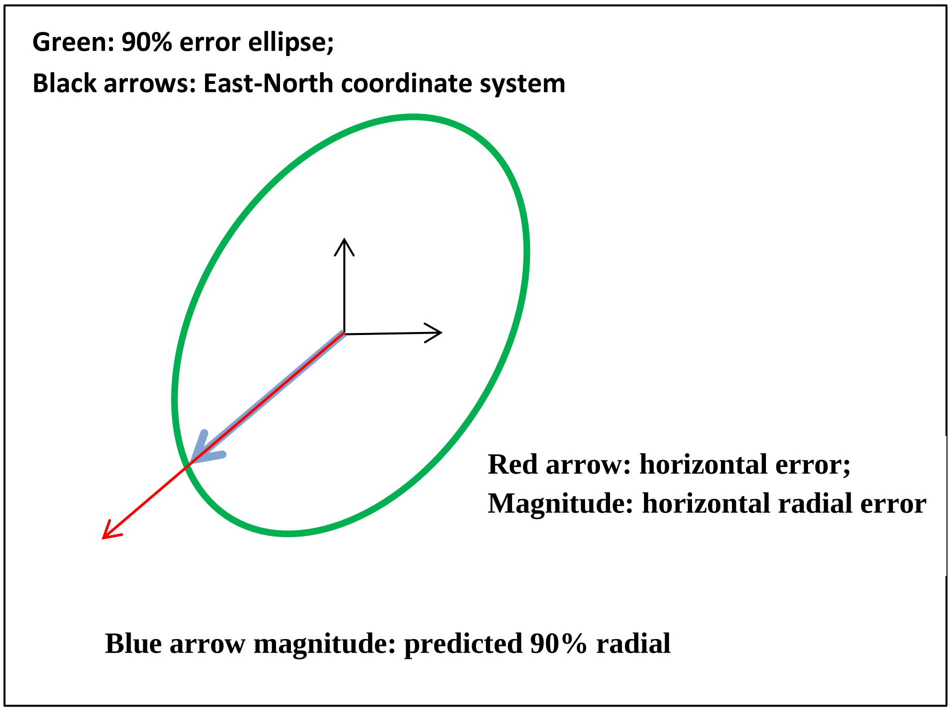


Figure 5.1.2-1: Example of horizontal error sample (red arrow) and its magnitude, horizontal radial error; corresponding radial vector of the 90% error ellipse (blue arrow), and its magnitude, predicted 90% radial; horizontal radial error excessively large in this particular example for clarity

Also, as implied by Figure 5.1.2-1, radial vectors (blue arrow) corresponding to independent samples of horizontal error (red arrow) will intersect the 90% error ellipse at different locations along its boundary and, correspondingly, will vary in magnitude assuming that the ellipse is non-circular, as is typical.

Note: As discussed earlier in this document, and as further detailed in TGD 2a (Predictive Statistics), an error ellipse, or more generally an error ellipsoid, is generated from an error covariance matrix and a specified level of probability.

<u>**Specification and Validation**</u>

The specification of predicted accuracy corresponds to the requirement that the Geolocation System either directly provides reliable predicted accuracies (error covariance matrices) associated with each extracted geolocation, or alternatively, provides support data for a "down-stream" application to generate such reliable predicted accuracies.

The formal specification of the above requirement and its validation correspond to (normalized) horizontal error samples that successfully satisfy each of the following three conditions:

1. At least yy1 % of the n corresponding samples of horizontal radial error are **less than** or equal to their corresponding predicted 99% radial
2. At least yy2 % of the n corresponding samples of horizontal radial error are **less than** or equal to their corresponding predicted 90% radial
3. At least yy3 % of the n corresponding samples of horizontal radial error are **greater than** or equal to their corresponding predicted 50% radial

The above conditions address three specific probability levels for practicality – 99, 90, and 50%. This approach does not attempt to assess the probability distribution over the entire range of errors, as this is impractical using a reasonable number of samples. Also, horizontal error samples are assumed independent and approximately identically distributed, where the qualifier "approximately" is consistent with error covariance matrices that vary somewhat over extracted geolocations, as expected.

The percentages yy1 – yy3 in the above predicted accuracy requirement are specified in TGD 2c as both a function of the number of independent samples n that are available as well as an assumed (specified) predicted accuracy fidelity for the Geolocation System, simply characterized as "high", "medium", or "low" for convenience, and soon defined below.

For convenience, the predicted accuracy requirement for a Geolocation System can include the specific level of predicted accuracy fidelity as a requirement instead of the explicit values for yy1-yy3, given that the latter can always be obtained from TGD 2c and that the (minimum) number of applicable samples is typically unknown when the specification is written.

<u>Definition of predicted accuracy fidelity</u>

The nominal value for "high" predicted accuracy fidelity corresponds to error covariance matrices (not actual errors) that are characterized as having standard deviations down their main diagonals (e.g., σ_x) that are within approximately $\pm$5% of their true but unknown values. The nominal value for "low" predicted accuracy fidelity corresponds to error covariance matrices that are characterized as having standard deviations down their main diagonals that are within approximately $\pm$35% of their true but unknown values. See TGD 2c for a precise definition of predicted accuracy fidelity and Section 5.5.3.1 of this document for additional background regarding the form for the precise definition.

By definition, high predicted accuracy fidelity is a proper subset of low predicted accuracy fidelity, i.e., all error covariance matrices that contain standard deviation within $\pm$5% of their true values are also within $\pm$35% of their true values. Of course, the other direction is not true, i.e., most error covariance with standard deviation that are within $\pm$35% of their true values are not within $\pm$5% of their true values.

The use of a specified level for predicted accuracy fidelity is realistic and supports practical, but thorough, specification and validation of predicted accuracy requirements. The use of different levels of predicted accuracy fidelity recognizes that no Geolocation System has perfect *a priori* error modeling, and that the degree of approximation varies across Geolocation Systems.

For example, if a Geolocation System corresponds to the NSG's use of same-pass stereo commercial satellite imagery, predicted accuracy is typically specified as "high" since the system is intended for a high-value mission and utilizes data collected by a specific sensor platform with a proven CONOPs. At the other end of the "spectrum", predicted accuracy fidelity may be specified as "low" for a new tactical sensor with highly varied collection environments and corresponding error sources with relatively unknown statistical characteristics.

If there were an unlimited number of error samples, and if predicted accuracy fidelity were perfect, the required percentages corresponding to the three conditions for specification/validation would approach the following: $yy1 \rightarrow 99$, $yy2 \rightarrow 90$, and $yy3 \rightarrow 50$ %. However, these percentages always correspond to lower values due to the finite number of (normalized) horizontal error samples available and to non-perfect predicted accuracy fidelity.

For example, if $n = 100$, and if high predicted accuracy fidelity is applicable, $yy1 = 95$, $yy2 = 81$, and $yy3 = 39$ %, which make the conditions for the validation of predicted accuracy reasonable, assuming that validation is appropriate. The fewer the number of samples of (normalized) horizontal error available and the lower the desired predicted accuracy fidelity, the lower these percentages become and the less difficult it is to meet all three of the conditions (99, 90, and 50%). On the other hand, and as explained in TGD 2c, the lower the number of available samples, the higher the probability that validation passes for an actual predicted accuracy fidelity that is at a category lower than that desired (specified), e.g., "medium" instead of "high".

Generic Example of Validation

The validation of the above predicted accuracy requirements is illustrated in Figure 5.1.2-2 for a general Geolocation System, where each of the lines correspond to one the three conditions discussed earlier. For example, the line with the higher slope corresponds to the condition that at least yy1% of the horizontal radial errors are less than their corresponding predicted 99% radial, i.e., at least yy1% of the horizontal radial error samples must be below the line. The correspondence between lines and conditions are detailed in TGD 2c – the higher the condition's corresponding probability level, the greater the slope of the line.

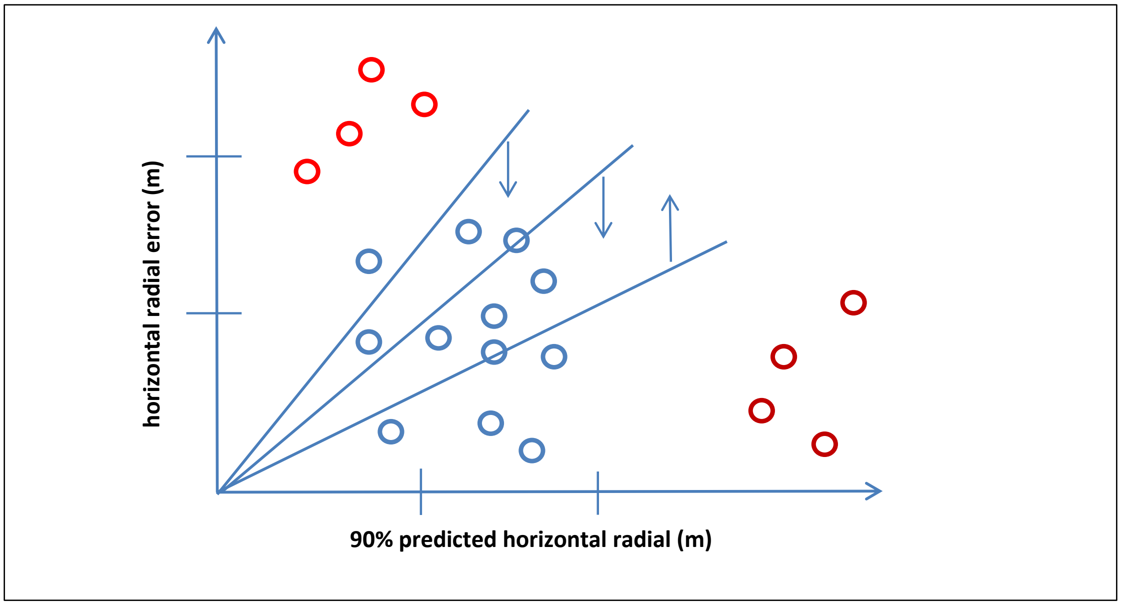


Figure 5.1.2-2 Conceptual graphic of the three probability-level normalized error tests (conditions) for the validation of predicted horizontal accuracy (all test lines blue); blue circles more prevalent than illustrated for clarity of plot

Note that the above figure's y-axis corresponds to horizontal radial errors and its x-axis to horizontal radials at the 90% predicted accuracy level, aka predicted 90% radials; thus, the middle line corresponds to a slope of 1.

Successful validation essentially ensures that extreme horizontal radial errors, the red circles (optimistic) and the dark red circles (pessimistic) in the figure, do not occur. The two lines with the larger slopes address optimistic predicted accuracies, and the line with the smallest slope addresses pessimistic predicted accuracies. Without the use of the latter line and the corresponding third (50%) condition, validation of predicted accuracy would essentially and mistakenly occur if predicted error covariance matrices were routinely computed as too large. Recall that the lower line corresponds to the condition that at least yy3 % number of horizontal radial error samples are above it, not below it.

<u>Specific Example of Validation</u>
Based on the above description, predicted accuracy requirements and their validation are also plot friendly. This is illustrated in Figure 5.1.2-3 using 100 independent samples of horizontal (radial) error.

The specified requirement for reliable predicted accuracies was validated as having been met: more than yy1=95% of the horizontal radial error samples were below the magenta line, more than yy2=81% were below the blue line, and more than yy3=39% were above the red line.

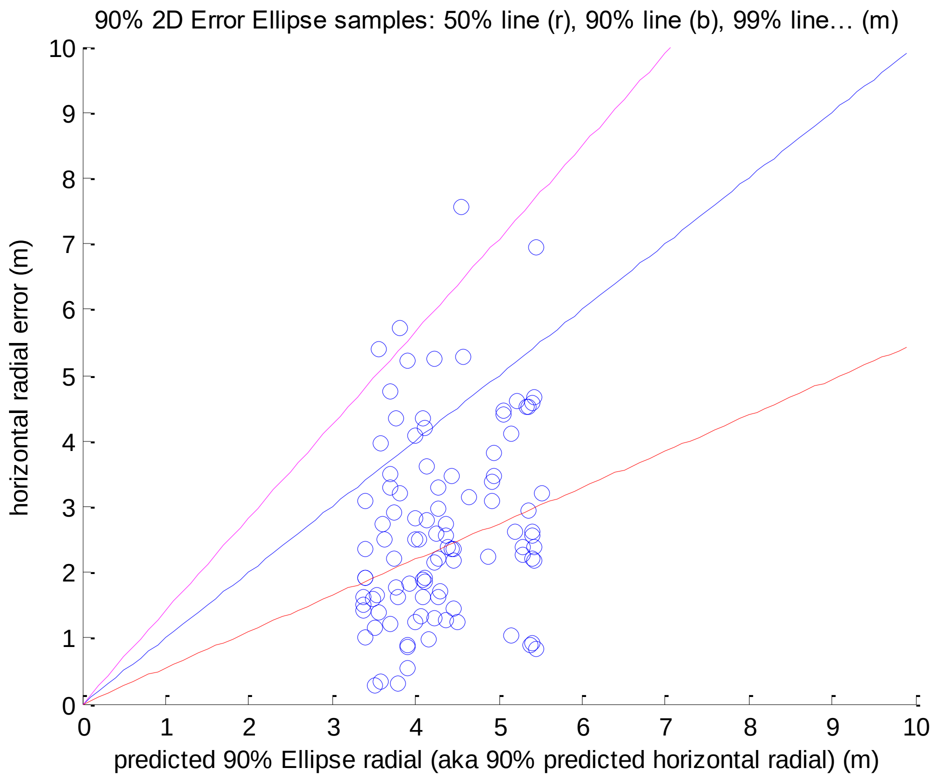


Figure 5.1.2-3: Graphical representation of the three normalized error tests corresponding to horizontal errors; horizontal radial error versus corresponding predicted 90% radial; 100 i.i.d. error samples, and high predicted accuracy fidelity – validation of predicted accuracy requirement successful

The above example is based on simulated data. Specifically, samples of error are consistent with true error covariance matrices which differ somewhat over the 100 samples. In addition, the computed error covariance matrices supplied by the extraction process differ from the true error covariance matrices, consistent with high predicted accuracy fidelity. That is, samples of error used in the validation process are consistent with true error covariance matrices. On the other hand, corresponding error covariance matrices supplied by the extraction process and also used in the validation process contain standard deviations that differ from their true counterparts by +/- 5%.

5.1.3 Summary

The specification and validation of accuracy requirements and the specification and validation of predicted accuracy requirements were described in Sections 5.1.1 and 5.1.2, respectively. The corresponding processes are practical, realistic, and thorough. Further details for each of these processes are provided in TGD 2c. Details include the recommended number of error samples for each process, as well as guidance on how to specify the actual values for the requirements (e.g. the actual numeric value for $CE90_{spec}$) that may be of use to NSG organizations that are responsible for the development and/or operations of a specific Geolocation System. The probability level XX for accuracy requirements is specifiable as equal to 95, 90, or 50%. Similarly, the confidence level YY is specified independently and as equal to 95, 90, or 50%. Reference [6] also provides an “easy-to-read” summary of TGD 2c.

Pseudo-code is available

Pseudo-code (MATLAB) is also included in TGD 2c that performs the entire validation process for accuracy requirements given specific accuracy and validation requirements as input, i.e., specific values for XX, $CEXX_{spec}$, YY, as well as a vector of n error samples. The pseudo-code outputs “pass” or “fail” results, as is applicable, and includes corresponding details, including plots similar to Figure 5.1.1-1. The pseudo-code also more generally implements the validation of accuracy requirements corresponding to horizontal errors, and/or vertical errors, and/or spherical (3d) errors.

Similarly, pseudo-code is also included that performs the entire validation process for predicted accuracy requirements given a specific predicted accuracy requirement as input, i.e., the category of predicted accuracy fidelity (“high”, “medium”, or “low”) as well as a vector of n error samples and their corresponding error covariance matrices. The category of predicted accuracy fidelity that is input corresponds to the nominal values for the tolerances, yy1, yy2, and yy3, that are embedded in the pseudo-code and that can be modified, if so desired. The pseudo-code outputs “pass” or “fail” results, as is applicable, and includes corresponding details, including plots similar to Figure 5.1.2-3. The pseudo-code also more generally implements the validation of predicted accuracy requirements corresponding to horizontal errors, and/or vertical errors, and/or spherical (3d) errors.

5.1.3.1 Additional comments

Appendix D presents additional comments in support of Sections 5.1, 5.1.1, and 5.1.2 that provide for a more complete overview of specification and validation of accuracy and predicted accuracy.

5.1.4 The External Data Challenge

The above performance validation and verification procedures are applicable to NSG self-generated data. If data origins and/or value-added processing are external, such as that associated with commodity data, crowd-sourcing, and outsourcing, procedures have to be modified in order to best deal with limited pedigree, accuracy, and quality assessment data. This is a significant and relatively new challenge, and is discussed in Section 5.12 of this document.

5.2 Statistical Error Model Overview

As discussed previously in Section 4, an appropriate statistical error model is recommended as applicable to virtually all accuracy and predicted accuracy applications in the NSG. A statistical error model is also more general than that applied in Section 5.1 which addressed the specification and validation of accuracy and predicted accuracy.

At the top-level, a statistical error model statistically describes the nx1 error vector ϵX corresponding to an nx1 state vector X as summarized in Figure 5.2-1, a repeat of Figure 4.3-1 for easier context.

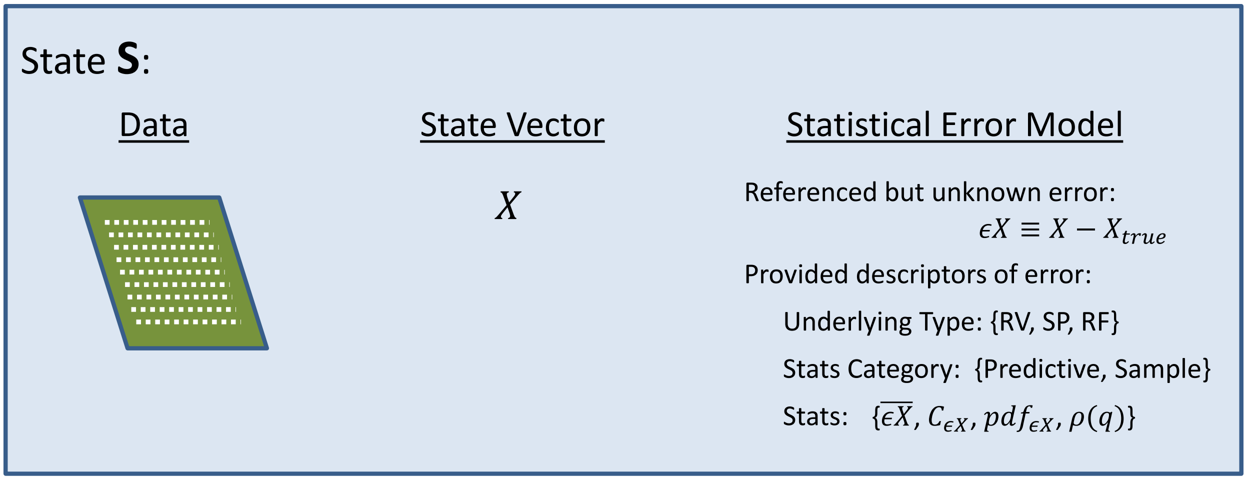


Figure 5.2-1: The statistical error model

More specifically, the statistical error model contains the following:

- Type of error representation
  - Random Vector (RV)
  - Stochastic Process (SP)
  - Random Field (RF)
- Category of statistics
  - Predictive
  - Sample
- Statistics
  - Mean-value, $\overline{\epsilon X}$, an $(nx1)$ vector
  - Covariance, $C_{\epsilon X}$, an (nxn) matrix
  - Optional correlation function $\rho(q)$, more specifically, a strictly positive definite correlation function (spdcf)
    - If stochastic process, q is scalar (e.g. delta time)
    - If random field, q is $mx1$ (e.g. delta m-dimensional spatial location)
  - Optional probability distribution or probability density function, $pdf_{\epsilon X}$
    - A Gaussian (normal) probability distribution or probability density function is already completely described by the above mean-value and covariance

Variations or "instances" of the above top-level statistical error model make-up the "tool box" for error modeling.

Note: throughout both this document and various TGD 2 documents, the explicit reference to errors ("ϵ") may be removed from statistic names/symbols for convenience, .e.g., $C_{\epsilon X} \rightarrow C_X$.

The statistical error model's "Type of error representation" is specified as either a Random Vector (RV), Stochastic Process (SP), or Random Field (RF). A $nx1$ random vector contains n random variables, and both a stochastic process and a random field consist of collections of random vectors. These various representations are discussed in detail in Section 5.3, and their accompanying statistics are tailored to the type of representation.

The statistical error model's "Category of statistics" is specified as either predictive or sample:

- Predictive statistics correspond to the mathematical modeling of assumed *a priori* error characteristics
- Sample statistics correspond to actual samples of the error.

The mean value for error is almost always zero for predictive statistics and typically not specified; if a non-zero error was predicted, it would simply be subtracted from X prior to further processing and hence, become zero. Also, the probability distribution need not be specified unless probabilities are to be assigned. When not specified but needed, it is typically assumed a Gaussian or Normal probability distribution.

Regardless the error model's category of statistics, the error covariance matrix describes the expected magnitude of the error vector ϵX and the inter-relationship of error among its n components. The error covariance matrix is assumed valid, i.e., positive definite, and hence, invertible. The techniques presented in these technical guidance documents ensure the generation/assembly of valid error covariance matrices. In addition, the error covariance matrices are to be realistic as possible, i.e., reasonably close to their corresponding true but unknown error covariance matrices, as discussed in TGD 2a.

The above state vector X and its corresponding error vector ϵX (Figure 5.2-1) are representative or "symbolic", in that, for an actual system's module, they may consist of a collection of separate and independent state vectors and their errors. They may also be a concatenation (stacking) of individual yet related (correlated) state vectors, i.e., $X = [X_1^T \quad .. \quad X_m^T]^T$, where $X_i, i = 1,..,m$, is of dimension $n_i x1$, and superscript T indicates vector transpose. If so, there is also a corresponding concatenation of error vectors $\epsilon X = [\epsilon X_1^T \quad .. \quad \epsilon X_m^T]^T$. Correspondingly, X and ϵX are termed the "multi-state vector" and the "multi-state vector error", respectively.

Section 5.3 now goes on to detail the types of error representation specifiable by the statistical error model (RV, SP, and RF). Examples are provided, including the corresponding predictive statistics and their metric values for these specific examples. Following that, Section 5.4 further details the statistical error model's category of statistics (predictive, sample) and the corresponding statistics themselves. It also provides an introduction to the key statistic of the error model – the error covariance matrix, defined in detail in Section 5.5.

5.3 Types of Error Representation: Random Vector, Stochastic Process, Random Field

As outlined in Section 5.2, a statistical error model's type of error representation corresponds to either a random vector, stochastic process, or random field. The error to be represented corresponds to an error

ϵX in a state vector X, although simply referenced to as a random vector ϵX or a collection of such random vectors below without reference to the state vector itself.

An overview regarding each type of representation is now given in order to provide more context for the remainder of this document. Some familiarity with probability and random variables is beneficial, with [15] and [12] good references as well as various TGD 2 documents. Graphics-based examples of each type of representation are also presented in the next sections, Sections 5.3.1 and 5.3.2, for more intuitive understanding.

A random vector (RV) contains from 1 to n components, each a random variable. A realization of a RV corresponds to specific values for its components and is associated with a given event, aka "trial" or "experiment". Multiple realizations are assumed independent, i.e., their corresponding components are uncorrelated. That is, given the value of one realization provides no additional information regarding the value of another realization. Important descriptive statistics of a RV are its mean (vector) value and the error covariance matrix about the mean. These statistics can be predictive or sample-based. Most statistics we deal with in this document are predictive. Because we are dealing with errors, we describe the random vector as ϵX, $nx1$. If it has more than one component, i.e., $n > 1$, the components (random variables) can be correlated between all of their possible pairs. This is termed "intra-state vector correlation".

A stochastic process (SP) is a collection of random vectors (RV), parameterized by a 1d vector q, typically time, i.e., parameterized by $q \equiv t_i$, $i = 1, .., m$, where m corresponds to the number of random vectors in the collection. For a given realization of the stochastic process, each of the individual random vectors ϵX_i correspond to a specific value $q = t_i$ and an arbitrary pair of random vectors are correlated, typically as a function of delta time between them. Correspondingly, although $\rho(q)$ is the general notation for the correlation function, $\rho(\Delta q) = \rho(\Delta t)$ is actually applicable. Furthermore, when specific random vectors ϵX_i and ϵX_j are identified, $\rho(\Delta t_{ij})$ is applicable, where delta time $\Delta t_{ij} = |t_i - t_j|$.

In general, there are n random variables (components) in ϵX_i, although many stochastic processes are simply scalar stochastic processes, i.e., $n = 1$. If $n > 1$, there can actually be multiple correlation functions corresponding to various subsets of the n components, although this is typically not the case.

If the statistics for the various random vectors ϵX_i are invariant over time, the stochastic process is termed (wide-sense) stationary and the correlation function is $\rho(\Delta t_{ij})$ as assumed above. If non-stationary, the correlation function is $\rho(t_i, t_j)$, a function of the actual times, not (just) their difference. An example of a stochastic process is the time series of sensor position error (3d) in satellite-based image metadata. It has three components, a mean value of zero, and can be modeled as approximately stationary in many instances, although not required. If the random vectors ϵX_i are collected into one large random vector ϵX, the temporal correlation between the various ϵX_i is also termed "inter-state vector" correlation.

A random field (RF) is an extension of a stochastic process parameterized by an N-dimensional vector q, instead of a 1 dimensional vector q. A typical application corresponds to N=2 or N=3, with q a horizontal or three-dimensional position on or near the earth's surface, e.g., $q \equiv X_i$, where $X_i = [x \quad y \quad z]^T{}_i$. Also, N=4 typically corresponds to three components of position and one of time. Two random vectors from the same realization of a random field are spatially correlated. For example, and for N=2, random vector ϵX_i corresponds to the horizontal location X_i, random vector ϵX_j corresponds to horizontal location X_j, and the two random vectors are correlated, typically as the function $\rho(\Delta X_{ij})$, assuming that the random field is (wide-sense) homogeneous, the equivalent of (wide-sense) stationary for a stochastic process.

Note:

(1) Regarding symbology, N corresponds to the number of spatial dimensions in a random field, n corresponds to the number of elements or components in a random vector. The use of n and N instead of n and N, respectively, corresponds to the formers' use in the convenient characterization of random fields as ND RF (nd), e.g., 2D RF (1d).

(2) A random variable can also be considered a random vector with the number of components or elements n=1. A stochastic process can also be considered a random field with the number of "spatial" dimensions N=1.

(3) The random vectors described in this document correspond to errors ϵX, and, therefore, are sometimes termed "random error vectors".

(4) A state vector estimate X can also be considered a random vector in that $X = X_{true} + \epsilon X$, where its mean-value is the deterministic quantity $X_{true} + \overline{\epsilon X}$, typically equal to $X_{true} + 0 = X_{true}$. X_{true} is the true but deterministic and typically unknown value of the state vector. The error covariance matrix of the state vector estimate X is equal to the covariance matrix of its error, $C_{\epsilon X}$.

5.3.1 Example for the direct comparison between types of representations

Hypothetical realizations corresponding to an RV, SP, and RF are presented in Figure 5.3.1-1, the SP at discrete times and the RF at discrete horizontal locations (N=2). All three sets of realizations correspond to two error components: ϵx and ϵy, i.e., 2d random vectors ($n = 2$). Let us term each set of realizations a "case".

For each case, realizations are independent (uncorrelated) from one another. For example, one realization of a random vector could correspond to the errors in an estimator's solution for a 2d geolocation, and another realization to another extraction from the same estimator for the same geolocation but using an independent set of measurements (errors) with the same predictive statistics as the first; and hence, the same predictive statistics for solution error.

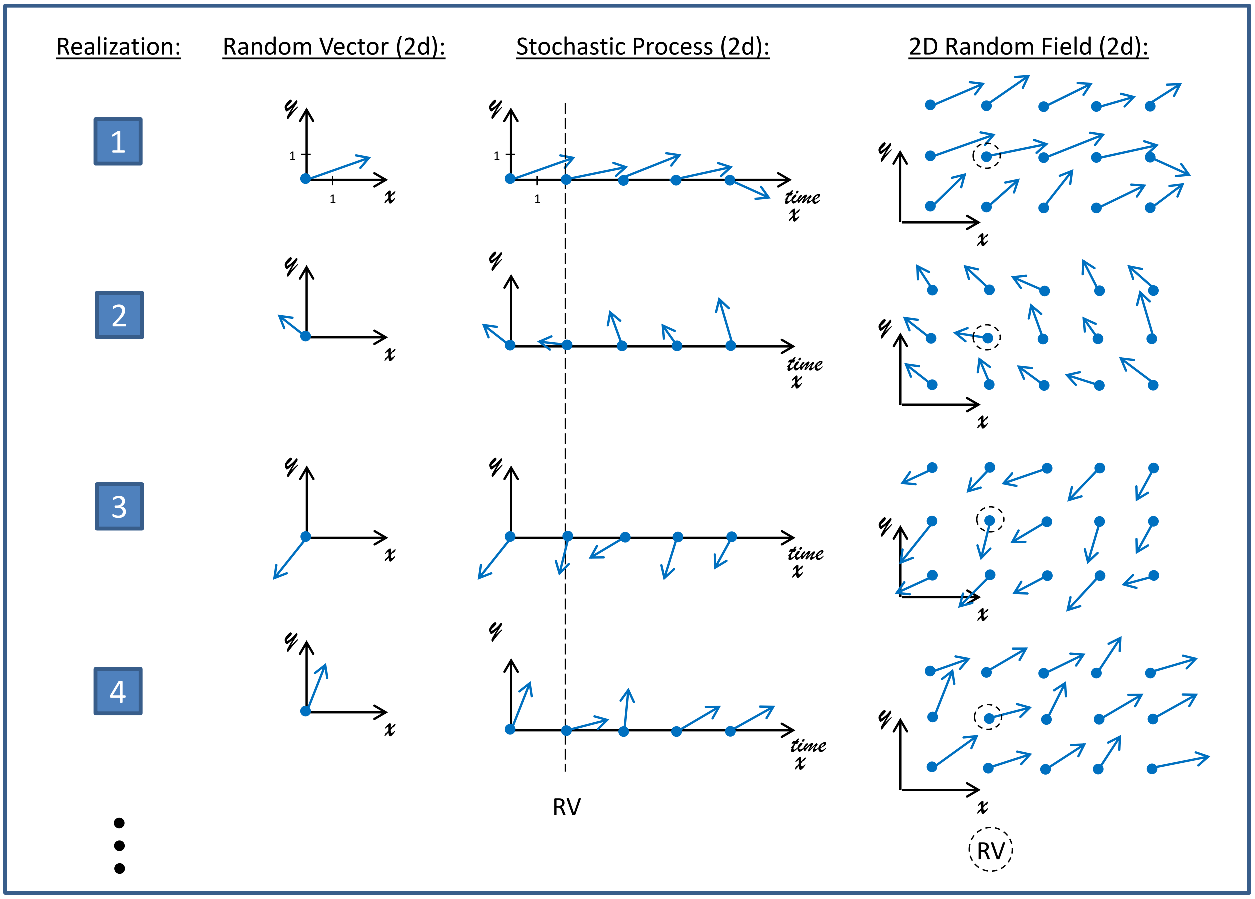


Figure 5.3.1-1: Multiple Realizations for a two-component RV, SP, and 2D RF

Figure 5.3.1-2 presents the corresponding predictive statistics for each case, where the various realizations were generated consistent with these predictive statistics. For all three cases, both ϵx and ϵy have a mean-value of zero and a standard deviation of 2 meters. These error components ϵx and ϵy are also uncorrelated, i.e., there is zero intra-state vector correlation. The corresponding error covariance matrix is diagonal with $2^2 = 4$ meters-squared down the diagonals.

In addition, the inter-state vector (temporal) correlation for the SP is modeled as a decaying exponential in delta time, with time constant TC=100 seconds. The inter-state vector (horizontal position) correlation for the RF is modeled as a product of two decaying exponentials, one in delta x-position and one in delta y-position, with distance constants of 150 m and 100 m, respectively. The cross-covariance as a function of delta time or delta position is also termed the covariance function, and equal to the common error covariance matrix at delta equal to zero.

Random Vector (2d):

$$\boldsymbol{\epsilon X} = \begin{bmatrix} \boldsymbol{\epsilon x} \\ \boldsymbol{\epsilon y} \end{bmatrix}$$

$$\overline{\boldsymbol{\epsilon X}} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix}$$

$$\boldsymbol{C_{\epsilon X}} = \begin{bmatrix} \boldsymbol{\sigma_{\epsilon x}^2} & \boldsymbol{\rho_{\epsilon x \epsilon y}\sigma_{\epsilon x}\sigma_{\epsilon y}} \\ \boldsymbol{\rho_{\epsilon x \epsilon y}\sigma_{\epsilon x}\sigma_{\epsilon y}} & \boldsymbol{\sigma_{\epsilon y}^2} \end{bmatrix} = \begin{bmatrix} \mathbf{4} & \mathbf{0} \\ \mathbf{0} & \mathbf{4} \end{bmatrix}$$

Intra-state Correlation $\boldsymbol{\rho_{\epsilon x \epsilon y}} = \mathbf{0}$

Inter-state Correlation n/a

Stochastic Process (2d):

$$\boldsymbol{\epsilon X(t)} = \begin{bmatrix} \boldsymbol{\epsilon x(t)} \\ \boldsymbol{\epsilon y(t)} \end{bmatrix}$$

$$\overline{\boldsymbol{\epsilon X(t)}} = \overline{\boldsymbol{\epsilon X}} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix}$$

$$\boldsymbol{C_{\epsilon X(t)}} = \boldsymbol{C_{\epsilon X}} = \begin{bmatrix} \mathbf{4} & \mathbf{0} \\ \mathbf{0} & \mathbf{4} \end{bmatrix}$$

Inter-state correlation:

$$\boldsymbol{\rho(\Delta t)} = \boldsymbol{e^{-|\Delta t|\frac{1}{100}}}, \boldsymbol{\Delta t} = \boldsymbol{t_i - t_j}$$

Cross-covariance:

$$\boldsymbol{C_{\epsilon X(t_i)\epsilon X(t_j)}} = \boldsymbol{C_{\epsilon X_{ij}}} = \boldsymbol{\rho(\Delta t)\, C_{\epsilon X}} = \begin{bmatrix} \boldsymbol{4e^{-|\Delta t|\frac{1}{100}}} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{4e^{-|\Delta t|\frac{1}{100}}} \end{bmatrix}$$

2D Random Field (2d):

$$\boldsymbol{\epsilon X(q)} = \begin{bmatrix} \boldsymbol{\epsilon x(q)} \\ \boldsymbol{\epsilon y(q)} \end{bmatrix}, \boldsymbol{q} = \begin{pmatrix} \boldsymbol{x} \\ \boldsymbol{y} \end{pmatrix}$$

$$\overline{\boldsymbol{\epsilon X(q)}} = \overline{\boldsymbol{\epsilon X}} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix}$$

$$\boldsymbol{C_{\epsilon X(q)}} = \boldsymbol{C_{\epsilon X}} = \begin{bmatrix} \mathbf{4} & \mathbf{0} \\ \mathbf{0} & \mathbf{4} \end{bmatrix}$$

Inter-state correlation: $\boldsymbol{\rho(\Delta q)} = \boldsymbol{e^{-|\Delta x|\frac{1}{150}}\, e^{-|\Delta y|\frac{1}{100}}}, \ \boldsymbol{\Delta q} = \boldsymbol{q_i - q_j} = \begin{bmatrix} \boldsymbol{\Delta x} \\ \boldsymbol{\Delta y} \end{bmatrix} = \begin{bmatrix} \boldsymbol{x_i - x_j} \\ \boldsymbol{y_i - y_j} \end{bmatrix}$

Cross-covariance: $\boldsymbol{C_{\epsilon X(q_i)\epsilon X(q_j)}} = \boldsymbol{C_{\epsilon X_{ij}}} = \boldsymbol{\rho(\Delta q) C_{\epsilon X}} = \begin{bmatrix} \boldsymbol{4e^{-|\Delta x|\frac{1}{150}}\, e^{-|\Delta y|\frac{1}{100}}} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{4e^{-|\Delta x|\frac{1}{150}}\, e^{-|\Delta y|\frac{1}{100}}} \end{bmatrix}$

Figure 5.3.1-2: Corresponding Error Model predictive statistics

5.3.2 Examples for further insight

For further insight, additional and higher-fidelity examples are now provided corresponding to a RV, SP, and RF.

The first example corresponds to a two-component RV and gives further insight into the meaning of intra-state (component) correlation. In Figure 5.3.2-1, there are two sets of 200 independent realizations of the RV, generated (simulated) consistent with the common predictive statistics for each set. The blue dots correspond to the RV predictive statistics of Figure 5.3.1-2. The red dots correspond to the same statistics except that intra-state correlation was changed from zero to a relatively high positive value with correlation coefficient $\rho_{\epsilon x \epsilon y} = 0.9$. For a given realization, whatever the value of ϵx, the corresponding value for ϵy is expected to be similar, born-out by the 45 degree "red-line" of dots corresponding to the samples. This degree of intra-state correlation is not uncommon for 2d errors associated with measurements from a "stand-off" sensor.

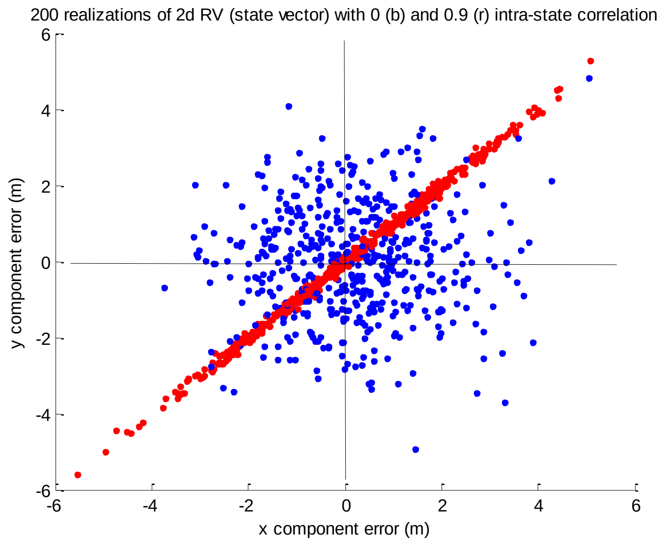


Figure 5.3.2-1: Independent realizations of a RV (2d): uncorrelated error components (blue), correlated error components (red)

Figure 5.3.2-2 corresponds to three independent realizations of a SP consisting of one-component ϵx only, with a mean value of zero and a standard deviation of 2 m. The temporal correlation was specified as in Figure 5.3.1-2 corresponding to a time constant TC=100 sec, but applicable to ϵx only. Note that sensor position metadata corresponding to commercial satellite imagery exhibits this general type of behavior. Figure 5.3.2-3 corresponds to one realization of the same SP, but with essentially zero temporal correlation (TC=1 sec) for comparison – note the high frequency variation of the realization over time.

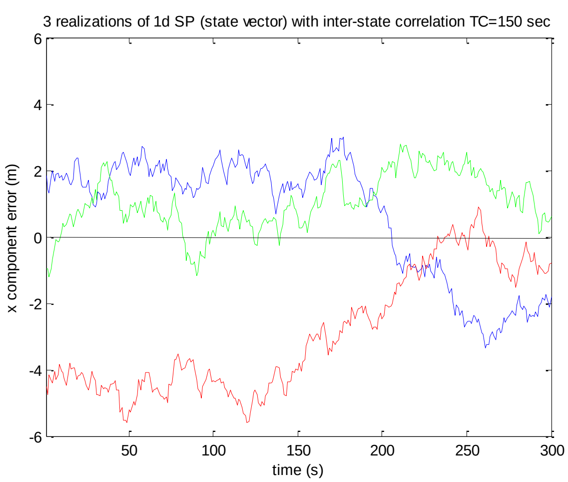


Figure 5.3.2-2: Three independent realizations of a SP (1d)

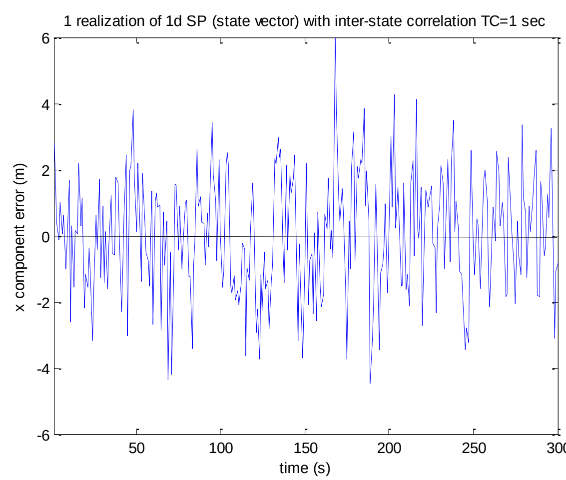


Figure 5.3.2-3: One realization of a SP (1d), no temporal correlation

Figure 5.3.2-4 corresponds to one realization of a 2D RF (1d), with one error component ϵz, and represented using a heat chart. The predictive statistics correspond to a mean value of zero, a standard deviation of 10 meters, and spatial correlation represented as $\rho(\Delta X) = e^{-|\Delta x|/19.5} e^{-|\Delta y|/19.5}$.

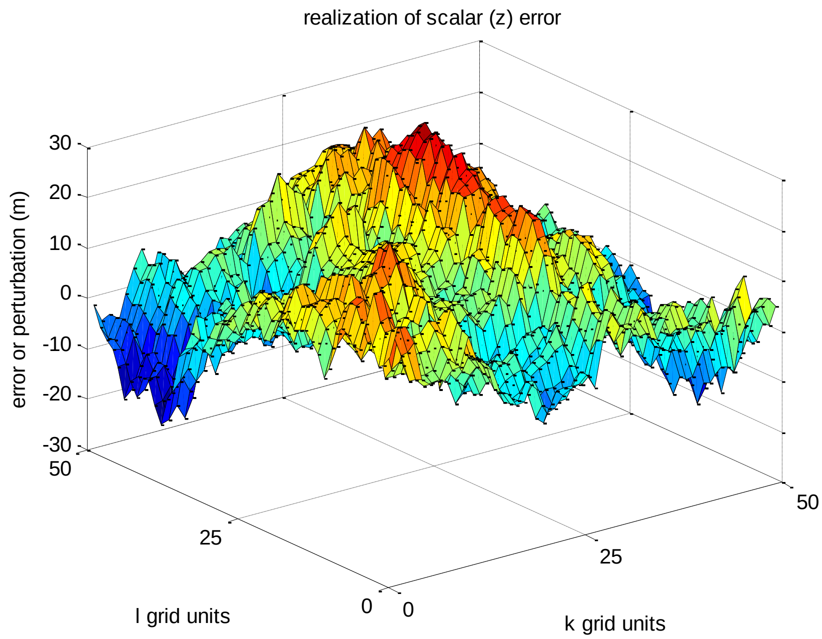


Figure 5.3.2-4: One Realization of a 2D RF (1d)

This RF could represent vertical Digital Elevation Model (DEM) error spatially correlated across a local horizontal plane, although both the specified standard deviation and spatial correlation function are hypothetical and for the purposes of illustration only. If the DEM was extracted using a stereo pair of images, inter-state (spatial) correlation is due to the common effects of image metadata errors across the pixels in the image/location in the horizontal plane. Note that in the figure, distance between grid points is 1 m in each horizontal direction.

In the above examples, the SP was (wide-sense) stationary (predictive statistics invariant across time) and the RF was (wide sense) homogeneous, an extension of stationarity to multiple dimensions. Non-stationarity and non-homogeneity may also be applicable in some cases, as discussed in some of the TGD 2 documents.

<u>Use of a probability distribution in simulation</u>

For the simulation of errors, identification of an assumed probability distribution, or equivalently, a probability density function $pdf_{\epsilon X}$, is always required in addition to the nominal predictive statistics, i.e., mean-value, error covariance matrix, and correlation function $\rho(q)$ or spdcf if a stochastic process or a random field. Correspondingly, the ubiquitous Gaussian probability distribution was selected for the simulation of all of the errors in this section. More specifically, each random vector, whether associated with a collection of random vectors in a stochastic process or a random field or not, was simulated consistent with a (multi-variate) Gaussian probability distribution per the associated techniques detailed in TGD 2e (Monte-Carlo Simulation).

Selection of the Gaussian distribution for the simulation of errors in this section makes sense in that the Gaussian distribution reasonably approximates most errors associated with real data, is completely and conveniently characterized by its mean-value and error covariance matrix, and it is relatively easy to simulate. However, for other geolocation-related applications, identification of an assumed probability distribution may or may not be required. For example, it is required to compute error or confidence ellipsoids, but it is not required to implement a Best Linear Unbiased Estimator (BLUE), as discussed in TGD 2d (Estimators and QC).

5.3.3 Additional terminology and the inclusion of Correlated Error

In the previous subsection, realizations of a random vector (RV), stochastic process (SP), and random field (RF) were presented. They were simulated based on corresponding predictive statistics which included intra-state vector correlation and inter-state vector correlation, the latter due to temporal or spatial correlation.

The effects of these correlations were illustrated in the samples or realizations presented in Figures 5.3.2-1 through 5.3.2-4. Correlation or statistical interdependence is key to reasonable and realistic modeling of errors in the NSG. It can have a very large effect on the relationships between errors as illustrated in these figures and, correspondingly, on solution results for (extracted) geolocations. The latter is further illustrated in Table 5.3.3-1 for a stereo same-pass commercial satellite imaging system with nominal imaging geometry [8].

Table 5.3.3-1 The effect of temporal correlation between same-pass images on WLS solution predicted accuracies

pred acc	temporal correlation		
	0%	80%	99%
CE90 m	3.0	3.6	3.7
LE90 m	6.3	3.6	2.5

As illustrated in the table, the degree of temporal correlation between images has a relatively small effect on horizontal accuracy but a large effect on vertical accuracy due to the cancellation of similar sensor metadata (pose) errors between the two images in the calculation of x-parallax. The higher the correlation, the smaller the vertical errors and the better the LE90. A value of temporal correlation greater than or equal to 80% is typical.

Note: The errors and the imaging geometry for the above system are modeled slightly different than for the system discussed in Section 4.6.1 but the same general principles apply.

In summary, the concept of correlation is very important and, correspondingly, should be included as part of an overall “error lexicon” as described below.

Errors and corresponding terminology

In years past, definitions for categories of error were sometimes simply limited to bias error and random or uncorrelated error, or their combined effect. These are now augmented with the inclusion of "correlated error" in conjunction with the use of RV, SP, and RF representations as follows:

<u>Categories of error</u>

We define three general categories of error relevant to NSG accuracy and predicted accuracy: (1) bias error, (2) random error, and (3) correlated error. These are defined in detail in Appendix A and in TGD 1G (Glossary), and are further summarized in the paragraphs below.

<u>Error representation</u>

Various combinations of errors from the above three general categories of error are then represented as either a: (1) random vector (RV), (2) stochastic process (SP), or (3) random field (RF), as appropriate, i.e., there are three different types of error representation as illustrated earlier in Sections 5.3.1 and 5.3.2. The appropriate error representation is also identified in the corresponding statistical error model as detailed in Section 5.2. Also, recall that SP and RF error representations consist of collections of random vectors, and that a random variable can be considered a random vector consisting of one component. Therefore, a random vector is the key element for all types of error representation.

<u>Realizations</u>

A "realization" of an error representation corresponds to an independent "trial" or "event" or "experimental outcome" as illustrated earlier in Sections 5.3.1 and 5.3.2. Two random vectors (minus their mean-value) from two different realizations are uncorrelated or "independent" by definition.

<u>Mapping between error category and error representation</u>

The "mapping" between error category (bias, random, correlated) and error representation (RV, SP, RF) is summarized as follows:

- A bias error corresponds to the non-zero mean-value of an error representation
- A random error corresponds to a random vector in an error representation minus its mean-value
- A correlated error corresponds to a random error that is correlated (statistically similar) with other random errors in the same realization of an error representation.
    - Examples:
        - One component of a random error is correlated with a different component of the same random error (intra-state vector correlation).
        - One random error is correlated with different random errors in the same realization of a collection of random vectors in a stochastic process or a random field (inter-state vector correlation). In the examples of Section 5.3.1, such correlation was quantified as a decaying exponential that was a function of delta time or spatial distance.

- In years past, a correlated error was sometimes simply represented indirectly as the sum of a random error and a bias error. Thus, two errors were considered correlated if and only if they shared a common bias. This is too simplistic and inappropriate.

Returning to terminology in general, the relatively common term "systematic error" is neither a category nor a representation of error per se. It is a characteristic of them or an effect from them. For example, the errors represented by a stochastic process or random field appear systematic across time or space, respectively, due to temporal or spatial correlation, respectively. The error in a frame image-based sensor model's adjustable parameter for focal length has a scaling effect on extracted ground locations that is systematic – the closer the ground point to the image footprint's boundary, the larger the effect – see TGD 1G (Glossary) for further details.

It is also possible to "transform" the representation of a predictive error from a stochastic process or random field to a single random vector and vice versa. For example, sensor metadata errors may be represented initially as a stochastic process (SP) and output as such in a Collection module corresponding to a time history of images, and then adjusted later in a Value-Added Processing module. The latter includes image-specific corrections for all images as part of a large combined state vector with corresponding error covariance matrix from an estimation process (e.g., Weighted Least Squares). As such, the sensor metadata error is now more conveniently thought of and represented as a single random vector (RV) with many components.

Finally, note that, for sample statistics, a non-zero sample mean does not necessarily imply that the underlying error representation or process includes a bias; that is, a non-zero sample mean can simply be due to the lack of statistical significance due to too few samples or to samples that were not independent, e.g., limited to the same realization of a stochastic process.

5.4 Statistical Categories: Predictive and Sample

As discussed earlier in Section 5.2, a statistical error model's statistics are categorized as either predictive or sample. Predictive statistics are "modeled" statistics, in that they correspond to an *a priori* mathematical model or are the output of a computational process, like an estimator. They are in contrast to sample statistics, which are typically generated "off-line" from a set of sampled errors using corresponding "ground truth". Of course, there is interplay between the two types of statistics: predictive statistics affect system errors which are then occasionally sampled. And sample errors can be used to better refine the predictive statistics and underlying predictive error models.

TGD 2a (Predictive Statistics) presents predictive statistics in detail, and TGD 2b (Sample Statistics) presents sample statistics in detail.

The use of the predictive statistics is more prominent in this document, although sample statistics do play an important role in: (1) Validation and Verification testing, (2) empirical experimentation in support of the development of appropriate predictive statistics, (3) Monte-Carlo simulation of errors and their effects, and (4) evaluation of accuracy for products generated external to the NSG.

Note that predictive statistics are almost always associated with predicted accuracy. Sample statistics are typically associated with accuracy per se but can also be used to "tune" predictive statistics associated with predicted accuracy.

Sections 5.5 – 5.7 of this document now go on to present an overview of the key predictive statistic: the error covariance matrix. However, it should be noted that a sample-based error covariance matrix, and corresponding sample mean, CE, LE, etc., can also be generated when appropriate as detailed in TGD 2b (Sample Statistics). In addition, these sample-based statistics include corresponding confidence intervals, or their equivalent, essentially specifying the confidence in their computed values as a function of the number of independent samples.

Finally, regarding the computation of sample-based scalar accuracy metrics (CE, LE, etc.) in TGD 2b, order statistics are also detailed and recommended as they require no assumption regarding the corresponding probability distribution of errors.

5.5 The key Predictive Statistic: the Error Covariance Matrix

The error covariance matrix $C_{\epsilon X}$, or more conveniently termed C_X , is the "key ingredient" in any statistical error model. It contains significant information regarding errors. Correspondingly, in probability theory, the inverse of the error covariance matrix C_X^{-1} is termed the "information matrix".

As we will see later, C_X^{-1} is also termed the "weight matrix" when corresponding to the errors in measurements used by an optimal estimator - the "smaller" the covariance matrix, the "larger" its inverse, hence, the more information contained in the corresponding measurements and the more weight they have on the estimator's solution.

The error covariance matrix is more formally defined as follows:

<u>Single state vector</u>

Let X be an $nx1$ single state vector and let $nx1$ ϵX represent its corresponding errors, i.e., $\epsilon X = X - X_{true}$. Let C_X represent the state vector's nxn (symmetric) error covariance matrix:

$$C_X = E\{\epsilon X \epsilon X^T\} = E\left\{\begin{bmatrix} \epsilon x_1^2 & \epsilon x_1 \epsilon x_2 & \cdot\cdot & \epsilon x_1 \epsilon x_2 \\ \epsilon x_2 \epsilon x_1 & \epsilon x_2^2 & \cdot\cdot & \epsilon x_2 \epsilon x_n \\ \cdot\cdot & \cdot\cdot & \cdot\cdot & \cdot\cdot \\ \epsilon x_n \epsilon x_1 & \epsilon x_n \epsilon x_2 & \cdot\cdot & \epsilon x_n^2 \end{bmatrix}\right\},$$

where $\epsilon X = [\epsilon x_1 \quad \epsilon x_2 \quad \cdot\cdot \quad \epsilon x_n]^T$, and where it is assumed that ϵX has a mean-value equal to zero, i.e., $E\{\epsilon X\} = \overline{\epsilon X} = 0_{n\times 1}$. The superscript T corresponds to vector transpose, and $E\{\,\}$ corresponds to expected value. Note that expected value is applicable to each entry in the covariance matrix, e.g. $E\{\epsilon x_1^2\}$. Also, if the mean-value is not zero, $C_X = E\{(\epsilon X - \overline{\epsilon X})(\epsilon X - \overline{\epsilon X})^T\}$.

Simply put, the above error covariance matrix quantifies the expected magnitude of each component of error and their interrelationships. The following is an example of an error covariance matrix, where X corresponds to a 3d geographic location:

$$C_X = E\left\{\begin{bmatrix} \epsilon x^2 & \epsilon x\epsilon y & \epsilon x\epsilon z \\ \epsilon y\epsilon x & \epsilon y^2 & \epsilon y\epsilon z \\ \epsilon z\epsilon x & \epsilon z\epsilon y & \epsilon z^2 \end{bmatrix}\right\} = \begin{bmatrix} \sigma_{\epsilon x}^2 & \rho_{\epsilon x\epsilon y}\sigma_{\epsilon x}\sigma_{\epsilon y} & \rho_{\epsilon x\epsilon z}\sigma_{\epsilon x}\sigma_{\epsilon z} \\ . & \sigma_{\epsilon y}^2 & \rho_{\epsilon y\epsilon z}\sigma_{\epsilon y}\sigma_{\epsilon z} \\ . & . & \sigma_{\epsilon z}^2 \end{bmatrix}, \text{ where}$$

the dots in the matrix represent symmetric entries, where the symbols σ^2 represents variance, σ represents standard deviation, and ρ represents correlation coefficient, discussed in more detail later. As presented in the general case earlier, the mean-value of error is also assumed equal to zero – almost always the case for predictive statistics because if it were not, its value could simply be subtracted from X such that the mean-value of error becomes zero, as desired. In addition, recall that the error in a state vector is considered a random vector (RV), as discussed in Section 4. The off-diagonal terms in the error covariance matrix correspond to intra-state vector correlation.

The following generalizes the above to multi-state vectors, presents a little more detail, and shares some of the context-dependent symbology.

Multi-state vector

More generally, let X_i be a $n_i x1$ state vector, and ϵX_i represent the corresponding errors in the state vector.

Let $X = [X_1^T \quad .. \quad X_m^T]^T$ represent the "stacked" $nx1$ multi-state vector corresponding to m individual state vectors and $\epsilon X = [\epsilon X_1^T \quad .. \quad \epsilon X_m^T]^T$ its $nx1$ multi-state vector error, where the superscript T indicates transpose. Let C_X represent the corresponding nxn multi-state vector (symmetric) error covariance matrix, where $n = n_1 + .. + n_m$:

$$C_X = E\{\epsilon X\epsilon X^T\} = E\left\{\begin{bmatrix} \epsilon X_1\epsilon X_1^T & \epsilon X_1\epsilon X_2^T & .. & \epsilon X_1\epsilon X_m^T \\ \epsilon X_2\epsilon X_1^T & \epsilon X_2\epsilon X_2^T & .. & .. \\ .. & .. & .. & .. \\ \epsilon X_m\epsilon X_1^T & \epsilon X_m\epsilon X_2^T & .. & \epsilon X_m\epsilon X_m^T \end{bmatrix}\right\} = \begin{bmatrix} C_{X1} & C_{X12} & .. & C_{X1m} \\ . & C_{X2} & .. & C_{X2m} \\ . & . & .. & .. \\ . & . & . & C_{Xm} \end{bmatrix}.$$

Note that C_{Xi} is the $n_i x n_i$ error covariance matrix for state vector i; C_{Xik} is the $n_i \times n_k$ error cross-covariance matrix between state vectors i and k, and E is the expected-value operator. The ϵX_i are random vectors, and the error covariance matrices C_{Xi} and C_{Xij} are typically predictive statistics based on assumed (but not necessarily specific) underlying probability distributions, and not sample statistics. The single dots "." in the above equation indicate symmetric entries (e.g., $C_{X21} = C_{X12}^T$), and the double dots ".." indicate "continue the pattern". C_X is a symmetric, positive definite matrix (strictly positive eigenvalues), i.e., invertible and a "valid" error covariance matrix. Note that because the above reference predictive errors, their mean values are assumed zero, i.e., $C_{Xi} = E\{(\epsilon X_i - \overline{\epsilon X_\iota})(\epsilon X_i - \overline{\epsilon X_\iota})^T\} = E\{(\epsilon X_i)(\epsilon X_i)^T\}$, and $C_{Xij} = E\{(\epsilon X_i - \overline{\epsilon X_\iota})(\epsilon X_k - \overline{\epsilon X_k})^T\} = E\{(\epsilon X_i)(\epsilon X_k)^T\}$.

The above error covariance formulation C_X is a natural representation for a SP or RF, which correspond to collections of RV's ϵX_i, $i = 1, .., m$, with error covariance matrix C_{Xi} and cross-covariance matrix C_{Xij}, $i, j = 1, .., m$. The cross-covariance matrices correspond to inter-state vector correlation. The formulation is also applicable to one or to an arbitrary collection of RVs, not necessarily associated with a SP or RF.

If we assume a Gaussian multi-dimensional distribution of errors, C_X specifies the entire joint probability distribution; however, assumption of a specific distribution is not required unless actual probabilities are to be assigned to various metrics.

5.5.1 Error Ellipsoids

An error ellipsoid is a graphical representation of the error covariance C_X and an intuitive representation of solution (predicted) accuracy. It displays, among other things, the directions of greatest and least expected solution error (magnitude). An error ellipsoid typically references a 3d error, either considered as corresponding to $3x1\ \epsilon X$ or a $3x1\ \epsilon X_i$, per the previous section, with corresponding error covariance C_X or C_{Xi} of the previous section. A Gaussian multi-variate distribution of errors is also assumed since the ellipsoid is associated with a specified probability, as detailed below.

The error ellipsoid presented in Figure 5.5.1-1 corresponds to a geographic 3d location error and was computed as a 90% (0.9p) error ellipsoid, which means that there is a 90% probability that the location (solution) error is within the ellipsoid. Alternatively, if the 90% error ellipsoid is centered at the target solution *X* instead of zero, there is a 90% probability that the true target location is within the ellipsoid. When centered at the target solution, the error ellipsoid is typically called a confidence ellipsoid. We are 90% confident that the true target location is within the 90% confidence ellipsoid.

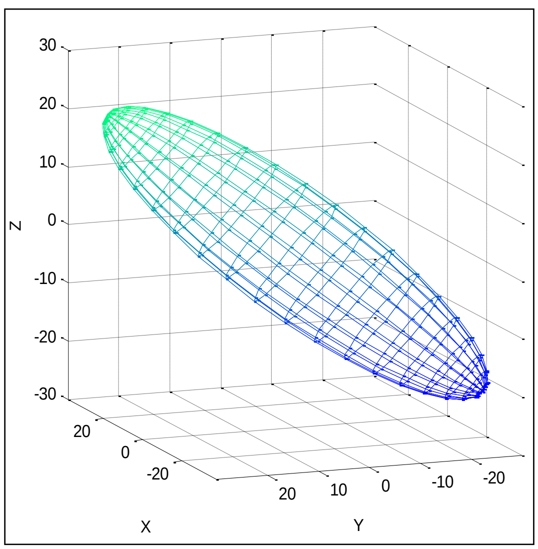


Figure 5.5.1-1: The 90% (0.9p) probability error ellipsoid corresponding and equivalent to C_X

The specific underlying error covariance matrix in this example is equal to:

$$C_X = \begin{bmatrix} \sigma_{\epsilon x}^2 & \rho_{\epsilon x \epsilon y}\sigma_{\epsilon x}\sigma_{\epsilon y} & \rho_{\epsilon x \epsilon z}\sigma_{\epsilon x}\sigma_{\epsilon z} \\ . & \sigma_{\epsilon y}^2 & \rho_{\epsilon y \epsilon z}\sigma_{\epsilon y}\sigma_{\epsilon z} \\ . & . & \sigma_{\epsilon z}^2 \end{bmatrix} = \begin{bmatrix} 10^2 & 0.75 \cdot 10 \cdot 12 & 0.95 \cdot 10 \cdot 9 \\ . & 12^2 & 0.8 \cdot 12 \cdot 9 \\ . & . & 9^2 \end{bmatrix}.$$

The various ρ designated in the error covariance matrix correspond to the intra-state correlation coefficient between the designated error components.

The general equation for an error ellipsoid (boundary) is presented in Figure 5.5.1-2:

The general equation for an error ellipsoid is given by: $\epsilon X^T C_X^{-1} \epsilon X = d^2$

For dim $n = 1$:

$$\epsilon X = \epsilon x$$

$$C_X = [E\{\epsilon x^2\}]$$

For dim $n = 2$:

$$\epsilon X = [\epsilon x \quad \epsilon y]^T$$

$$C_X = \begin{bmatrix} E\{\epsilon x^2\} & E\{\epsilon x \epsilon y\} \\ . & E\{\epsilon y^2\} \end{bmatrix}$$

For dim $n = 3$:

$$\epsilon X = [\epsilon x \quad \epsilon y \quad \epsilon z]^T$$

$$C_X = \begin{bmatrix} E\{\epsilon x^2\} & E\{\epsilon x \epsilon y\} & E\{\epsilon x \epsilon z\} \\ . & E\{\epsilon y^2\} & E\{\epsilon y \epsilon z\} \\ . & . & E\{\epsilon z^2\} \end{bmatrix}$$

Figure 5.5.1-2: Equation for the Error Ellipsoid

The value for d in the above equation is different for different desired levels of probability and dimension n: for a 90% level and 1D (line), 2D (ellipse), 3D (ellipsoid), d is equal to 1.64, 2.15, and 2.50, respectively. This also assumes a (multi-variate) Gaussian distribution of errors. See TGD 2a for 0.5. 0.9. 0.95, 0.99, and 0.999 probability levels, more significant digits for d, accommodation for an atypical non-zero mean-value, as well as the general equation for an arbitrary probability level. All error ellipsoids in this document correspond to 90% probability unless specifically designated otherwise.

Note, as mentioned previously, given the desired level of probability, an error ellipsoid and its corresponding error covariance matrix are equivalent. The error covariance matrix (along with the probability level or d) is used to compute the ellipsoid via the general equation of Figure 5.5.1-2. Although not as obvious, the error covariance matrix can also be derived from the corresponding error ellipsoid as detailed in TGD 2a.

5.5.2 Full error covariance matrix needed

The full error covariance matrix is needed for the statistical representation of both absolute and relative errors. This is best illustrated when an individual state vector corresponds to the 3d geographic location of a feature of interest, and the overall state vector corresponds to the concatenation of two state vectors corresponding to two different 3d features or locations.

Let the error covariance matrix for the first location correspond to:

$$C_{X1} = \begin{bmatrix} 20^2 & 0.98 \cdot 20 \cdot 10 & 0.90 \cdot 20 \cdot 10 \\ . & 10^2 & 0.90 \cdot 10 \cdot 10 \\ . & . & 10^2 \end{bmatrix} = \begin{bmatrix} 400 & 196 & 180 \\ . & 100 & 90 \\ . & . & 100 \end{bmatrix}$$, with units of meters-squared.

Figure 5.5.2-1 presents the corresponding and correct error ellipsoid, typical for a 3d location extracted from a stand-off EO imaging sensor. If instead of the full C_{X1}, assume that intra-state (component) correlations were ignored or set to zero instead of the correct values of 0.98 and 0.90, i.e., C_{X1} was replaced by its diagonal matrix counterpart for simplicity. The corresponding error ellipsoid is presented in Figure 5.5.2-2 – note the loss of correct expected magnitude and directionality of errors when the full C_{X1} is not used.

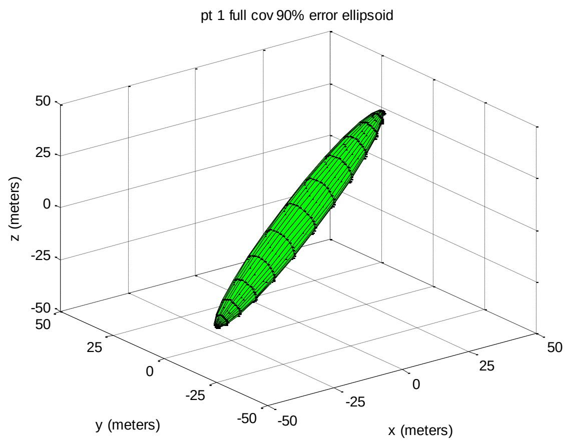


Figure 5.5.2-1: Correct Error ellipsoid for point 1

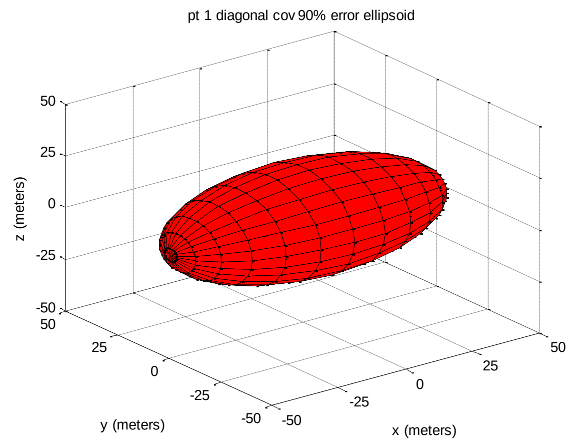


Figure 5.5.2-2: Incorrect Error ellipsoid for point 1

We assume a similar (but not exactly the same) error covariance matrix C_{X2} for another ground point extracted from the same image (plus DTED), with corresponding correct error ellipsoid presented in Figure 5.5.2-3.

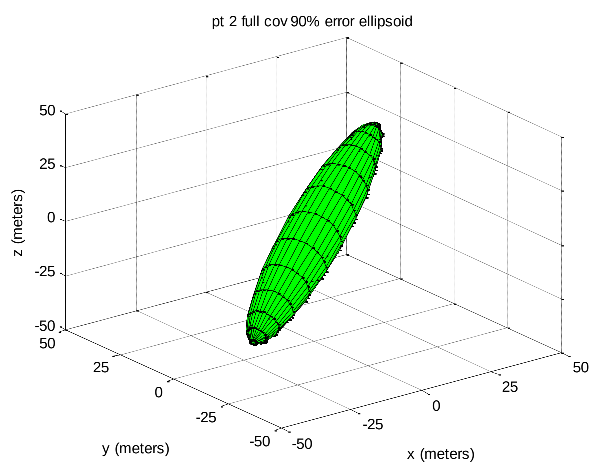


Figure 5.5.2-3: Correct Error Ellipsoid for point 2

We also assume a cross-covariance C_{X12} between the 3d errors at the two locations with common inter-state correlation of 0.9 across all components. The availability of C_{X1}, C_{X2}, and C_{X12} allows for computation of the relative error covariance matrix $relC_{X12}$, as documented in TGD 2a. This error covariance matrix corresponds to the relative error $(\epsilon X_1 - \epsilon X_2)$. Note that (detailed in TGD 2a):

$C_X = \begin{bmatrix} C_{X1} & C_{X12} \\ . & C_{X2} \end{bmatrix}$ (6x6), and $relC_{X12} = C_{X1} + C_{X2} - C_{X12} - C_{X21}$ $(3x3)$.

Figure 5.5.2-4 presents the corresponding 90% relative error ellipsoid for points 1-2 computed from $relC_{X12}$. Note its smaller size as compared to Figures 5.5.2-1 and 5.5.2-3, i.e., due to positive correlations: common 3d errors in the two locations cancel statistically with a resultant smaller error covariance and error ellipsoid.

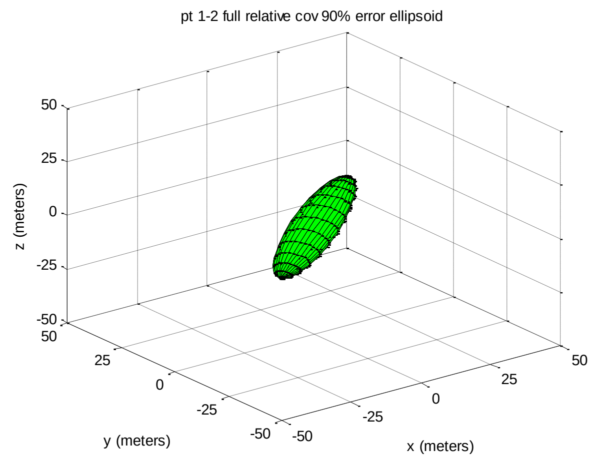


Figure 5.5.2-4: Correct Relative Error Ellipsoid for points 1-2

Finally, Figures 5.5.2-5 and 5.5.2-6 illustrate the incorrect 90% relative error ellipsoids for points 1-2 obtained if the full $6x6$ C_X contains only the correct diagonal blocks (no inter-state correlation, i.e., $C_{X12} = 0$), and only the correct diagonals (no intra-state or inter-state correlations, i.e., C_X a diagonal matrix), respectively.

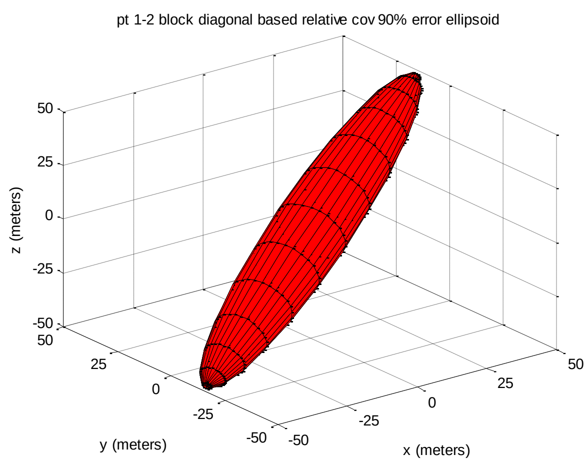


Figure 5.5.2-5: Incorrect Relative Error Ellipsoid (diagonal blocks)

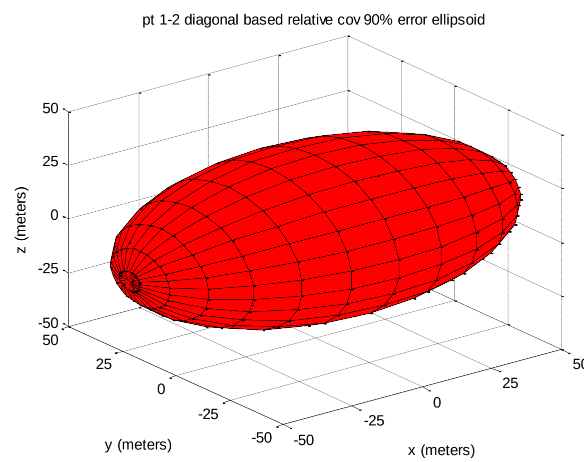


Figure 5.5.2-6: Incorrect Relative Error Ellipsoid (diagonals only)

See reference [4] for more details regarding the above example.

As illustrated in this subsection, the output and use of the correct multi-state vector error covariance matrix is critical for a correct and informed assessment of both absolute and relative accuracy, i.e., for reliable predicted absolute accuracy and for reliable predicted relative accuracy. Although presented for location errors specifically, this same concept and conclusions are applicable to an arbitrary multi-state vector error ϵX, made-up of various arbitrary individual state vector errors ϵX_i.

5.5.3 Additional applications of error covariance matrices and error ellipsoids

The previous (sub) sections of Section 5.5 illustrated inherent applications of error ellipsoids in support of operational decisions and analyses, such as visualizing the probability of error in the geolocation of a feature of interest in all directions relative to its surroundings. 3d error ellipsoids were illustrated for generality, but 2d error ellipses are frequently used in a similar manor.

Other such applications associated with the error covariance matrix and/or error ellipsoid are presented below and correspond to brief overviews of related content in TGD 2a (Predictive Statistics). In general, the applications are applicable to nxn error covariance matrices and to their error ellipsoids. However, for better intuitive understanding, examples below assume horizontal errors and corresponding $2x2$ error covariance matrices and error ellipses.

Related applications that do not necessarily support operational decisions, but do support other algorithms and procedures presented in these documents, are presented as well.

5.5.3.1 Comparison of error covariance matrices

Support of operational decisions and analyses

Some operational decisions may involve automated/automatic decisions regarding which estimate of a geolocation to use when multiple such estimates are available, typically computed previously using different sources of data. A candidate approach involves the automated/automatic comparison of their associated error covariance matrices, designated A and B, assuming that two estimates are applicable.

In general, the relationship $A < B$, or equivalently, $B > A$, involving error covariance matrices of the same size has numerous applications, such as those associated with the design of estimators (see Section 5.5.3.2). TGD 2a defines this relationship as the eigenvalues of the matrix $(B - A)$ being strictly positive.

TGD 2a also proves that the corresponding error ellipse for A is totally contained within the error ellipse for B, as illustrated in Figure 5.5.3.1-1; thus, demonstrating that the geolocation associated with error covariance matrix A should be selected instead of the geolocation associated with error covariance matrix B, given no other mitigating factors. This selection is easily performed by automatically computing the eigenvalues of the matrix $(B - A)$, a simple function call in most programming languages, such as MATLAB.

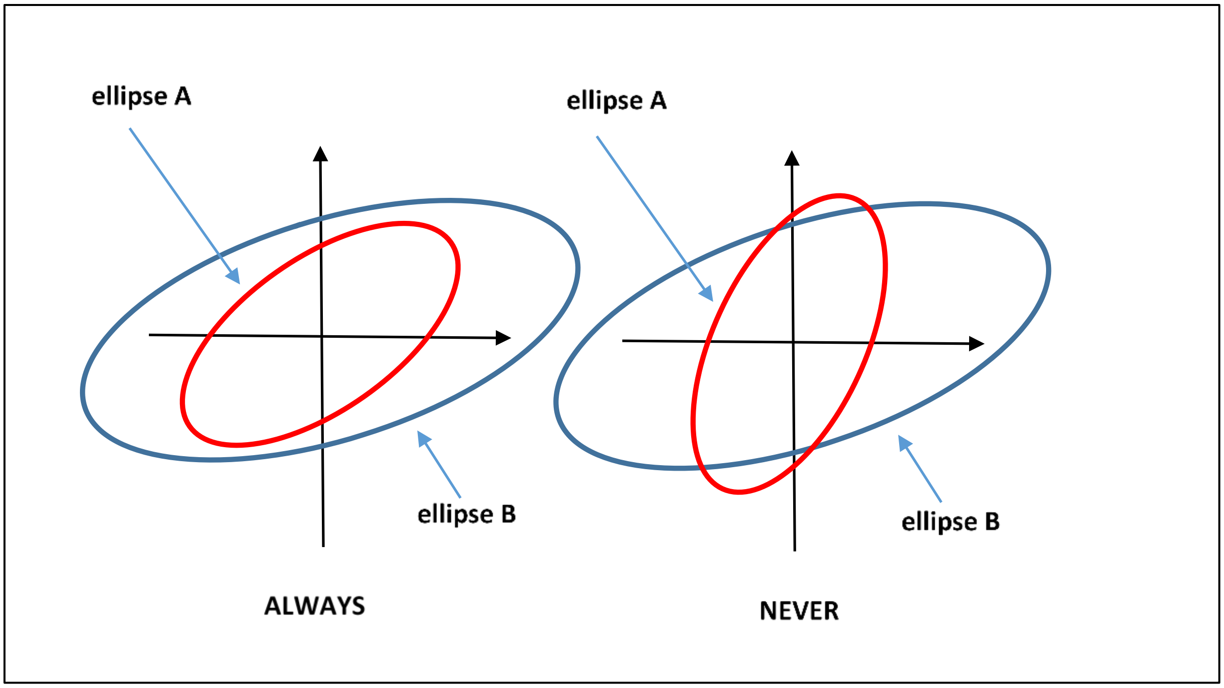


Figure 5.5.3.1-1: $B > A$ implies that the error ellipse A corresponding to error covariance matrix A is better than the error ellipse B corresponding to error covariance matrix B; probability or confidence levels of the ellipses are arbitrary as long as common

If $A \leq B$ instead, then the boundary of the ellipse A is interior to and/or on the boundary of the ellipse B. In general, it is also possible that neither $A \leq B$ nor $B \leq A$, in which case other automatic selection methods for the appropriate estimate of the geolocation can be implemented as discussed in TGD 2a.

Support of general analyses

The above relationships between error covariance matrices also enables the following inequality:

$sf_1 A \leq A_{true} \leq sf_2 A$, where

the scale factors satisfy $0 < sf_1 \leq 1 \leq sf_2$ and A is an *a priori* or computed error covariance matrix associated with an error ϵX, and A_{true} is its true but unknown counterpart. The scalar multiplication of the error covariance matrix A by a scale factor sf corresponds to the multiplication of each of its elements or components by sf, and hence, its standard deviations by $\sqrt{sf}$.

Figure 5.5.3.1-2 presents an example using corresponding error ellipses. The inequality allows us to bound the true but unknown error covariance matrix A_{true} (green ellipse) between two scalar multiples (blue dashed ellipses) of the given error covariance matrix A (blue ellipse).

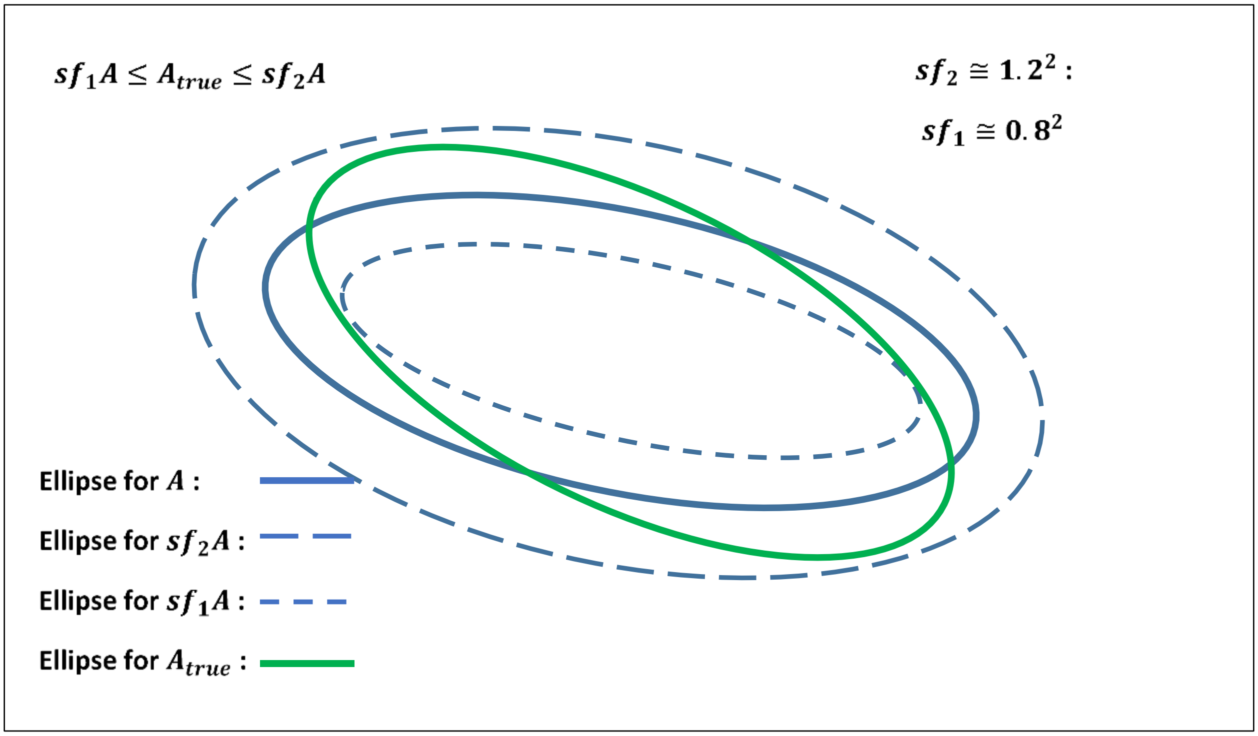


Figure 5.5.3.1-2: Bounding the true but unknown error covariance matrix illustrated using corresponding error ellipses; the probability levels of the solid error ellipses are arbitrary as long as common

The above inequality has many applications, including its central role in the formal definition for "predicted accuracy fidelity" - see Section 5.1.2 regarding the specification and validation of predicted accuracy for the informal and approximate definition of predicted accuracy fidelity which relies on the differences between the standard deviations of the true error covariance matrix and the supplied error covariance matrix. See TGD 2c (Specification and Validation) for the formal and precise definition of predicted accuracy fidelity.

Another application of the above inequality corresponds to its use in the new, robust, and recommended reference variance test for estimator QC – see Section 5.9.4.2 and TGD 2d (Estimators and their QA/QC) for more detail.

5.5.3.2 The Method of Covariance Intersection

Other applications of the error covariance matrix and/or error ellipsoid for operational decisions as well as for general analyses involve the unions and the intersections of error covariance matrices, as detailed in TGD 2a. A particularly important application involving the intersection of error covariance matrices corresponds to the Method of Covariance Intersection, illustrated by example in Figure 5.5.3.2-1.

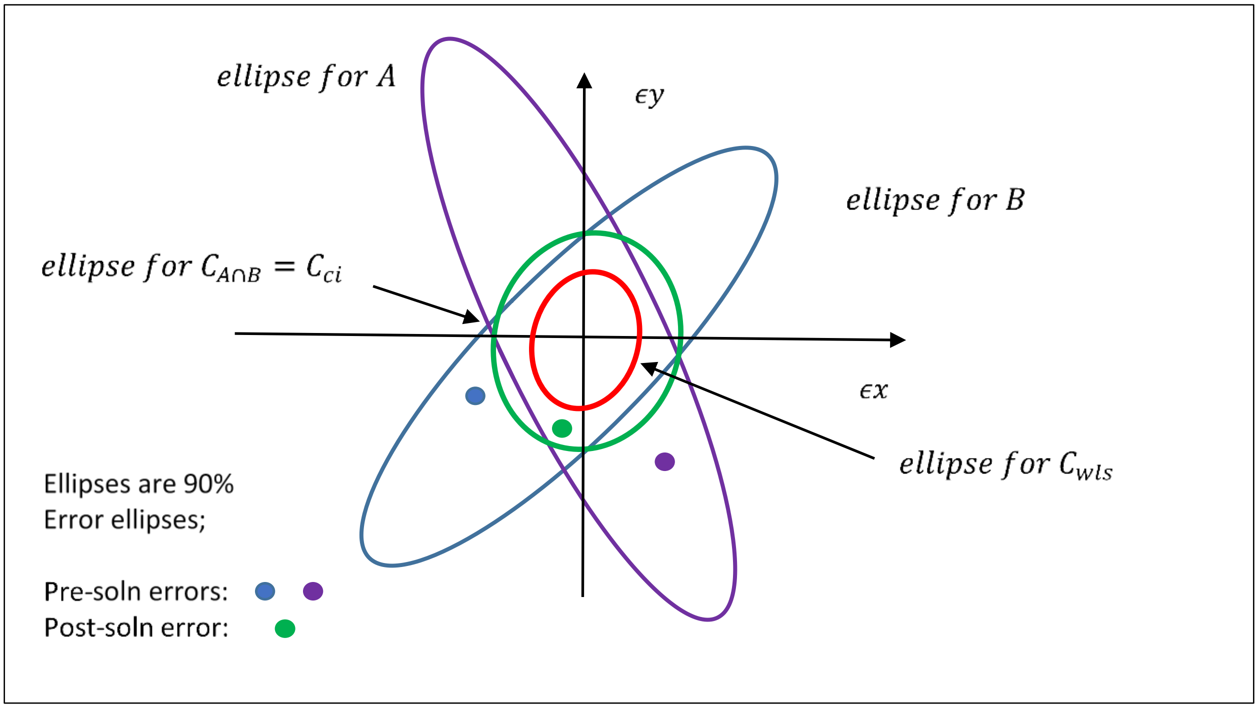


Figure 5.5.3.2-1: The Method of Covariance Intersection matrix; $C_{ci} = C_{A\cap B}$ is a practical upper bound for the true but unknown error covariance matrix C_{true} for the estimate X_{ci} based on two *a priori* estimates with unknown correlation of errors between them; the error in X_{ci} corresponds to the green dot; the more *a priori* estimates available, the more dramatic the results.

The Method of Covariance Intersection computes the best possible estimate X_{ci} for a geolocation X using two *a priori* estimates (or direct measurements) of the geolocation, termed X_a and X_b, such as two estimates based on measurements from a stand-off sensor taken on the same flight path. The error covariance matrices, A and B, for the two *a priori* estimates are assumed known, but their correlation of errors (cross-covariance matrix) is unknown, i.e. the corresponding *a priori* error model (predictive statistics) is incomplete.

The Method of Covariance Intersection also computes the error covariance matrix C_{ci} of its solution X_{ci} as the intersection of the two error covariance matrices, A and B, as detailed in TGD 2a. Figure 5.5.3.2-1 presents the corresponding pre-solution errors (blue and purple dots) and the post-solution error (green dot). The pre-solution errors are positively correlated in this example, but the degree of correlation is unknown to the solution process.

The error covariance matrix C_{ci} corresponds to the green 90% error ellipse in the figure. It bounds from above the true but unknown post-solution error covariance matrix C_{true}, i.e., $C_{true} \leq C_{ci}$. If a WLS solution were performed instead, it would unavoidably assume that the two initial estimates were uncorrelated, i.e., the correlation between the two *a priori* estimates was zero. Correspondingly, its solution would be the same as the solution based on the Method of Covariance Intersection, but its computed solution error covariance C_{wls} would be incorrect – optimistic and corresponding to the red ellipse in the above figure. Use of an optimistic error covariance matrix should always be avoided, as it usually leads to bad consequences.

The above was a simple example of the relatively new and important Method of Covariance Intersection, which can also be applied to multiple estimates of an arbitrary $nx1$ state vector with unknown correlation of errors. It yields the smallest possible conservative error covariance matrix. The technique can also be applied when the error covariance matrices for the *a priori* estimates (or direct measurements) are not known explicitly and are replaced by upper bounds.

Potential applications of the Method of Covariance Intersection also include the combination of multiple single-image ("mono") extractions or multiple multi-image extractions (MIGs) that are based on image measurements from the same sensor(s) with non-trivial but unknown sensor biases. These unknown sensor biases induce unknown correlation of errors between the multiple extractions.

5.6 Scalar Accuracy Metrics: Linear Error, Circular Error, and Spherical Error

Scalar accuracy metrics are ubiquitous across the NSG and are used to quantify location accuracy at a specified level of probability, assumed equal to 0.90 or 90%, if not specified explicitly. Scalar accuracy metrics can be either predictive statistics or sample statistics. The calculations of scalar accuracy metrics are detailed in TGD 2a for predictive statistics, and in TGD 2b for sample statistics.

The definitions of scalar accuracy metrics are presented below, along with an overview of their calculations as predictive statistics, which also assume a (multi-variate) Gaussian distribution of errors.

- The scalar accuracy metric Linear Error (LE) corresponds to a vertical error and is computed from the lower right $1x1$ portion of the full $3x3$ error covariance matrix C_X. LE corresponds to the length of a vertical line (segment) such that there is a 90% probability that the absolute value of vertical error resides along the line. If the line is doubled in length and centered at the target solution, there is a 90% probability the true target vertical location resides along the line.
- The scalar accuracy metric Circular Error (CE) corresponds to horizontal error and is computed from the upper left $2x2$ portion of the full $3x3$ error covariance matrix C_X. CE corresponds to the radius of a circle such that there is a 90% probability that the horizontal error resides within the circle, or equivalently, if the circle is centered at the target solution, there is a 90% probability the true target horizontal location resides within the circle.
- The scalar accuracy metric Spherical Error (SE), corresponds to 3d error and is computed from the full $3x3$ error covariance matrix C_X. SE corresponds to the radius of a 3D sphere such that there is a 90% probability that 3d error resides within, or equivalently, if the sphere is centered at the target solution, there is a 90% probability the true target location resides within the sphere.
- The above calculations for scalar accuracy metrics assumed a mean-value of error equal to zero, as is typical for predictive statistics. However, both TGD 2a and TGD 2b also account for non-zero mean-values in their calculation when applicable.

Note that we have assumed that the underlying x-y-z coordinate system is a local tangent plane system, i.e., x and y are horizontal components and z the vertical component. If not, the error covariance matrix must first be converted to correspond to such a system prior to computation of LE, CE, or SE.

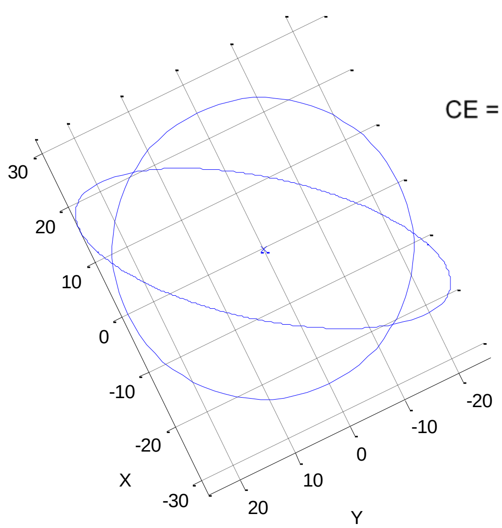
Scalar accuracy metrics are easy to understand and are in common use for military applications. Also, LE and CE are sometimes used together to form a "CE-LE cylinder" in preference over SE in order to represent 3d accuracy, as illustrated later.

LE, CE, and SE are also convenient approximations to various error ellipsoids (line, ellipse, and ellipsoid, respectively) that can also be generated from portions of the underlying $3x3$ error covariance matrix C_X. Like the scalar accuracy metrics, the error ellipsoids have an associated specified level of probability (default 90%). Unlike the scalar accuracy metrics (except LE), the error ellipsoids are equivalent to the underlying error covariance and not "approximations".

Figures 5.6-1 and 5.6-2 present examples of CE and a CE-LE cylinder, respectively, computed from the following underlying error covariance matrix:

$$C_X = \begin{bmatrix} \sigma_{\epsilon x}^2 & \rho_{\epsilon x \epsilon y}\sigma_{\epsilon x}\sigma_{\epsilon y} & \rho_{\epsilon x \epsilon z}\sigma_{\epsilon x}\sigma_{\epsilon z} \\ . & \sigma_{\epsilon y}^2 & \rho_{\epsilon y \epsilon z}\sigma_{\epsilon y}\sigma_{\epsilon z} \\ . & . & \sigma_{\epsilon z}^2 \end{bmatrix} = \begin{bmatrix} 10^2 & 0.75 \cdot 10 \cdot 12 & 0.95 \cdot 10 \cdot 9 \\ . & 12^2 & 0.8 \cdot 12 \cdot 9 \\ . & . & 9^2 \end{bmatrix}.$$

The figures include a corresponding error ellipse and ellipsoid, respectively, for comparison.



CE = 24.766 m

Circular area: 1926.93348 m²
Ellipse area: 1148.32767 m²

Equation for circle:

$$\epsilon x^2 + \epsilon y^2 = CE^2 = 24.8^2$$

Equation for 90% ellipse:

$$\epsilon X^T C_X^{-1} \epsilon X = d^2$$

$$\frac{\epsilon x^2}{\sigma_{\epsilon x}^2(1-\rho_{\epsilon x \epsilon y}^2)} + \frac{2\rho_{\epsilon x \epsilon y}\epsilon x \epsilon y}{\sigma_{\epsilon x}\sigma_{\epsilon y}(1-\rho_{\epsilon x \epsilon y}^2)} + \frac{\epsilon y^2}{\sigma_{\epsilon y}^2(1-\rho_{\epsilon x \epsilon y}^2)} = d^2$$

$$\frac{\epsilon x^2}{43.75} - \frac{\epsilon x \epsilon y}{35} + \frac{\epsilon y^2}{63} = 2.15^2$$

Figure 5.6-1: CE vs ellipse

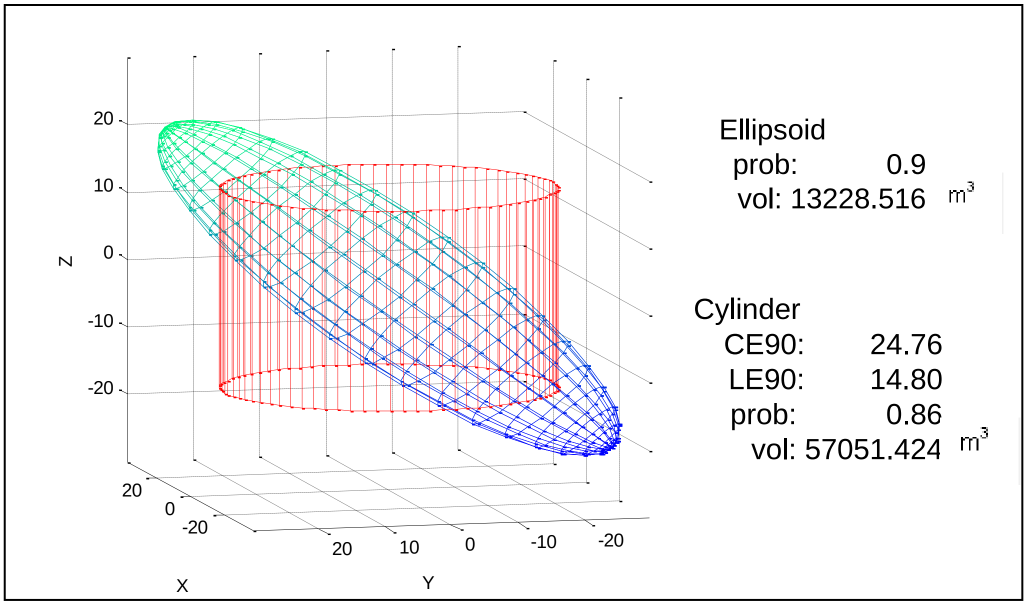


Figure 5.6-2: CE-LE cylinder vs 3D error ellipsoid

5.6.1 Desirable Characteristics of Scalar Accuracy Metrics

A desirable feature of scalar accuracy metrics is that they provide a natural representation of accuracy and a convenient summary of predicted accuracy. In fact, by definition, they have a specified probability of error associated with them and correspond to an easy to understand radial error (vertical, horizontal, or spherical). As mentioned earlier, 90% probability is the assumed default, but can be specified otherwise; for example, CE_95 corresponds to Circular Error at the 95% probability level.

Of course, scalar accuracy metrics refer to the absolute accuracy of a 3d location. In addition, scalar relative accuracy metrics (rel LE, rel CE, and rel SE) are also applicable and easily computed as detailed in TGD 2a as convenient summaries of predicted relative accuracy between two 3d locations.

Scalar accuracy metrics are convenient, one-number summaries of accuracy: easy to understand, and to picture. They are ubiquitous across the NSG; hence the need for standardized computation as detailed in TGD 2a for predictive statistics and TGD 2b for sample statistics. They are also tied to ordinance characteristics and essential for tactical operations.

5.6.2 Limitations of Scalar Accuracy Metrics

On the other hand, scalar accuracy metrics have significant limitations for the representation of predicted accuracy, as discussed in the next two subsections. Therefore, scalar accuracy metrics should supplement, not replace, the underlying error covariance matrix or its equivalent error ellipsoid.

5.6.2.1 Inefficiency and loss of information with scalar accuracy metrics

Use of the error ellipse by the military can allow for more precise operations than if CE were used instead. For example, a monoscopic target extraction using an image from a stand-off optical sensor will yield an elongated error ellipse in the horizontal plane, e.g. 10:1 ratio of semi-major to semi-minor axis, due to the low elevation angle of the line-of-sight (LOS) and/or external elevation uncertainty. Figure 5.6.2.1-1 presents an example of the elongated 90% error ellipse and corresponding CE.

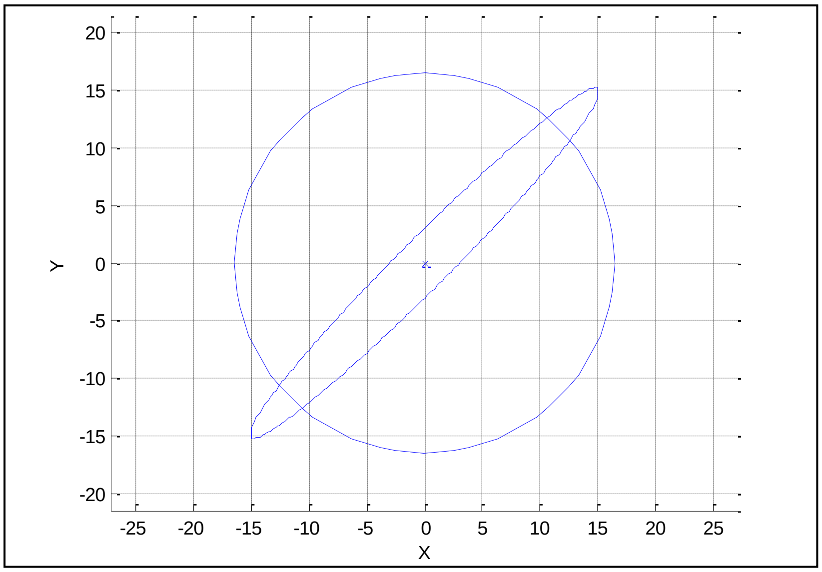


Figure 5.6.2.1-1: Error Ellipse and Corresponding CE circle

The CE equals 16.5 meters, with a corresponding area within the circle of 853 meters-squared. The area within the ellipse is 145 meters-squared. The ellipse and the circle contain the same 90% probability that the target's true horizontal location resides within, but the ellipse requires much less area than does the circle. Operational concentration on the area within the ellipse instead of the area within the circle may allow for smaller "search" area, limited collateral damage, etc.

Further technical details regarding the above example are as follows: The LOS and semi-major axis are on the same vertical plane. The underlying error covariance matrix had a standard deviation in both horizontal directions of sqrt(50.5), and a correlation coefficient of 0.98 between them.

The following is another example of inefficiency or loss of information, this time corresponding to SE or 90% Spherical Error. Figure 5.6.2.1-2 presents the sphere with radius SE versus the 3-D ellipsoid corresponding to the same error covariance matrix C_X presented in Section 5.6.

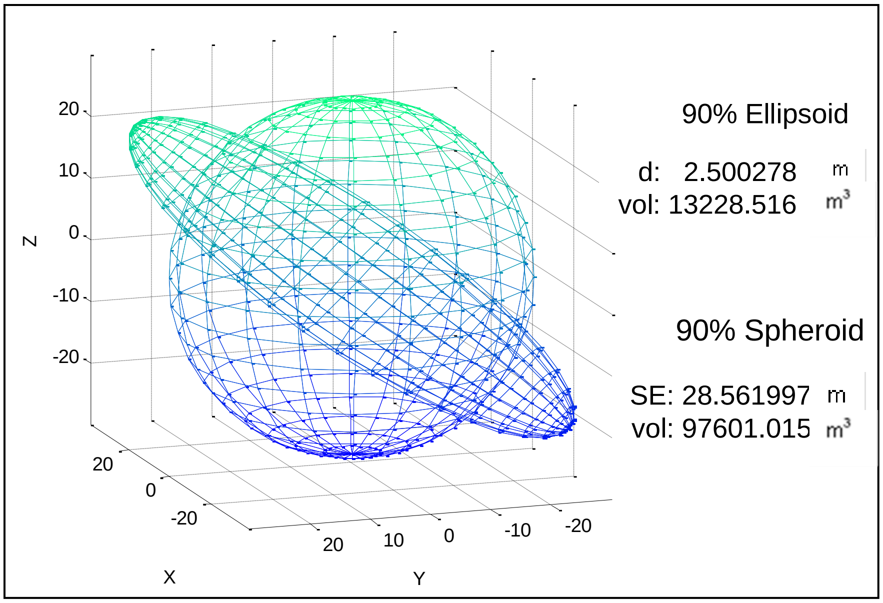


Figure 5.6.2.1-2: Error Ellipsoid versus corresponding SE spheroid

Note that the sphere requires over 7 times the volume (m^3) as does the ellipsoid to encompass the same probability. This is the price one pays for the simplicity and convenience of using one number to represent the six unique numbers contained in the error covariance or error ellipsoid. All sense of direction-dependent uncertainty is lost with SE and the other scalar accuracy metrics. This example also serves to illustrate one final point. In general, the error ellipsoid is the most "efficient" shape there is to represent a given amount of uncertainty – for dimension $n = 3$ it requires the least volume to enclose a desired level of probability as compared to all other shapes. More specifically, for dimension $n = 2$ it requires the least area as compared to rectangles, circles, or any other closed curve.

In summary, the error ellipsoid is preferred over scalar accuracy metrics for the graphical display of the error covariance matrix and the information contained within regarding the expected magnitude and interrelationships of error components.

5.6.2.2 Inferior fusion with scalar accuracy metrics

Fusion is a process that combines or relates different sources of information. A generic example is the best estimate of a 2d location given two independent estimates of that location along with their corresponding $2x2$ error covariance matrices, or equivalently, their corresponding error ellipses. Figure 5.6.2.2-1 illustrates this process, where the blue dots correspond to the individual estimates, the red triangle to the best estimate of the location using both estimates weighted by their corresponding error covariance matrices, the green diamond is the true location, and the red ellipse is the solution's error ellipse.

If the two independent estimates came with CE instead of the actual error covariance, their corresponding error covariance matrices are equivalent to the blue circles in Figure 5.6.2.2-2, and the "best estimate" would be the red triangle with corresponding and significantly larger error relative to truth (green diamond) than in Figure 5.6.2.2-1. Note that the use of CE corresponds to the loss of intra-state vector correlation.

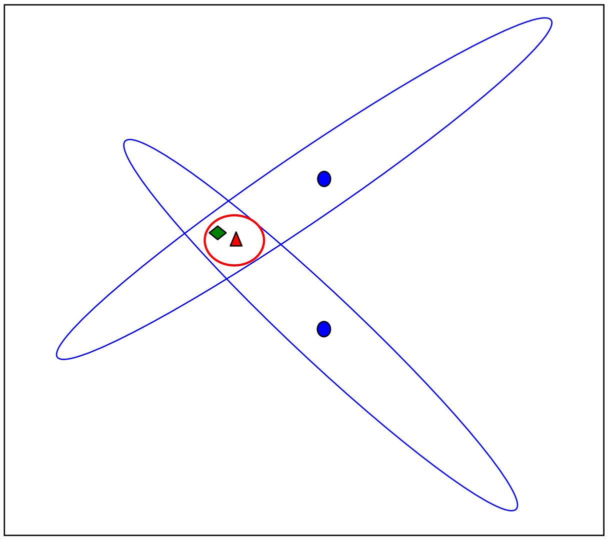


Figure 5.6.2.2-1: Optimal fusion based on error covariance

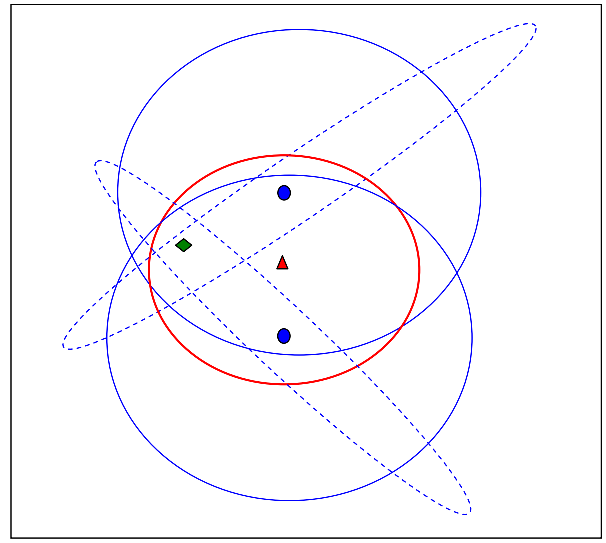


Figure 5.6.2.2-2: Inferior fusion based on CE

As illustrated above, appropriate fusion cannot take place without corresponding error covariance matrices. Reference [3] presents further details of the above example.

5.7 Representation/Dissemination of Error Covariance Matrices

A full multi-state vector error covariance matrix or its equivalent can be represented/disseminated in three general ways: Direct, "A matrix", and "Spdcf", as summarized below and detailed in TGD 2a (Predictive Statistics).

Recall from Section 5.5 of this document that the multi-state vector is represented as $X = [X_1^T \ \ .. \ \ X_m^T]^T$, its error as $\epsilon X = [\epsilon X_1^T \ \ .. \ \ \epsilon X_m^T]^T$, and the corresponding multi-state vector error covariance matrix as:

$$C_X = E\{\epsilon X \epsilon X^T\} = E\left\{\begin{bmatrix} \epsilon X_1 \epsilon X_1^T & \epsilon X_1 \epsilon X_2^T & .. & \epsilon X_1 \epsilon X_m^T \\ \epsilon X_2 \epsilon X_1^T & \epsilon X_2 \epsilon X_2^T & .. & .. \\ .. & .. & .. & .. \\ \epsilon X_m \epsilon X_1^T & \epsilon X_m \epsilon X_2^T & .. & \epsilon X_m \epsilon X_m^T \end{bmatrix}\right\} = \begin{bmatrix} C_{X1} & C_{X12} & .. & C_{X1m} \\ . & C_{X2} & .. & C_{X2m} \\ . & . & .. & .. \\ . & . & . & C_{Xm} \end{bmatrix}.$$

Let us assume that X and the multi-state vector error covariance matrix are to be disseminated and a subset subsequently assembled "down-stream" corresponding to three of the individual state vectors X_1, X_3, and X_5, as a specific example. This example not only serves for convenience of description, but is typical operationally. For example, if X corresponds to the solution for adjusted image support data in an image bundle adjustment of $m = 200$ images over a large area of interest, there are typically multiple downstream applications that use different subsets of these adjusted (registered) images in order to accurately extract ground points over their smaller area of interest. However, the bundle adjustment (Value-Added Processing module) must output the entire X and C_X in order to serve all of the applications.

The descriptions below do not include the corresponding multi-state vector X or its components X_1, X_3, and X_5 for convenience and the fact that bandwidth is dominated by the error covariance matrix. As a reminder, the down-stream application is only interested in X_1, X_3, and X_5 and their corresponding (full) error covariance matrix termed C_{X*}.

Direct method

Disseminate: $C_{X1}, C_{X12}, C_{X13}, .. , C_{X1m}, C_{X2}, C_{X23}, C_{X24}, .. , C_{X2m}, .. , C_{Xm}$.

Assembly example for state vector errors $\epsilon X_i,\ i = 1{,}3{,}5$: $C_{X*} = \begin{bmatrix} C_{X1} & C_{X13} & C_{X15} \\ . & C_{X3} & C_{X35} \\ . & . & C_{X5} \end{bmatrix}$.

"A matrix" method

Disseminate: $C_{X1}, A_1^2, C_{X2}, A_2^3, .. , C_{Xm-1}, A_{m-1}^m, C_{Xm}, A_m^{m+1}$.

Assembly example for state vector errors $\epsilon X_i,\ i = 1{,}3{,}5$: $C_{X*} = \begin{bmatrix} C_{X1} & C_{X1}(A_2^3 A_1^2)^T & C_{X1}\left(A_4^5 A_3^4 A_2^3 A_1^2\right)^T \\ . & C_{X3} & C_{X3}\left(A_4^5 A_3^4\right)^T \\ . & . & C_{X5} \end{bmatrix}$.

The "A matrix" method is compatible with a Kalman Filter (or smoother, with some modifications) that sequentially outputs A_i^{i+1} in addition to the usual X_i and C_{Xi}. A Kalman Filter is a sequential estimator, but its standard version (no "A matrix" capability), cannot generate the cross-covariance matrix C_{Xij}. This recommended capability is documented in TGD 2a.

Spdcf method

Disseminate: $C_{X1}, C_{X2}, .. , C_{Xm}$; and a few parameters defining the scalar-valued, strictly positive definite correlation function (spdcf), designated $\rho(\delta t)$, where δt can correspond to delta time or delta space, and can be a scalar or multi-dimensional. (δt_{ik} is alternatively designated Δt_{ik}).

Assembly example for state vector errors $\epsilon X_i\ i = 1{,}3{,}5$:

$C_{X*} = \begin{bmatrix} C_{X1} & \rho(\delta t_{13}) \cdot \left(C_{X1}^{1/2}\right)\left(C_{X3}^{1/2}\right) & \rho(\delta t_{15}) \cdot \left(C_{X1}^{1/2}\right)\left(C_{X5}^{1/2}\right) \\ . & C_{X3} & \rho(\delta t_{35}) \cdot \left(C_{X3}^{1/2}\right)\left(C_{X5}^{1/2}\right) \\ . & . & C_{X5} \end{bmatrix}$, where the superscript 1/2 indicates principal matrix square root, a symmetric matrix. (Note that if $C_{Xi} = C_{Xk}$, $\left(C_{Xi}^{1/2}\right)\left(C_{Xk}^{1/2}\right) = C_{Xi}$.)

The use of an spdcf in the above equation insures a valid error covariance matrix C_{X*}. A specific spdcf is selected based on desired correlation characteristics. There are numerous spdcf families, some of which are illustrated in Figure 5.7-1, and further detailed in TGD 2a. For a RF, they can also be assembled as isotropic (spatial direction independent) or anisotropic (spatial direction dependent), such as the product of two damped exponential spdcf, as illustrated in Figure 5.7-2.

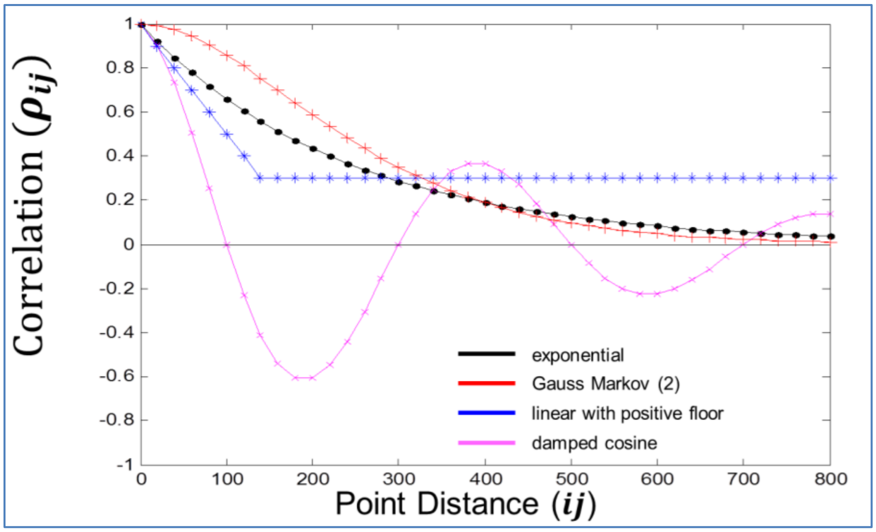


Figure 5.7-1: Families of spdcf

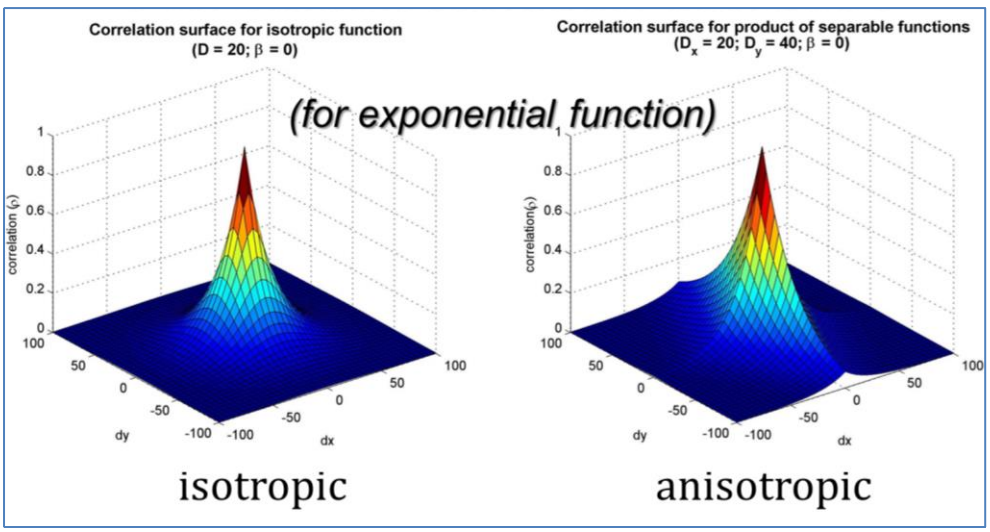


Figure 5.7-2: Isotropic and anisotropic spdcf for a 2D RF (see reference [10])

The term “strictly positive definite correlation function” or “spdcf” refers to the fact that the use of such a function in the assembly of C_{X*} insures a resultant positive definite error covariance matrix. On the other hand, use of a “positive definite correlation function” or “pdcf”, only insures a resultant positive semi-definite error covariance matrix. Also, an spdcf’s functional-value itself is not necessarily strictly positive as demonstrated in Figure 5.7-1.

If the multi-state vector error covariance matrix corresponds to a stationary SP (or homogeneous RF), the above representation is exact (assuming based on an *a priori* error model). If non-stationary, it is typically an approximation, although the assembled error covariance matrix is guaranteed valid (positive definite).

The spdcf method compresses the corresponding multi-state vector error covariance matrix – only the diagonal blocks and the few parameters describing the spdcf need be disseminated/retained. See TGD 2a and [4, 5] for a more complete description of the spdcf method.

Summary

In terms of general representation/dissemination, the direct method typically corresponds to a full error covariance matrix corresponding to an estimate (adjustment) of a multi-state vector, such as generated by a batch Weighted Least Squares (WLS) estimator. The multi-state vector is typically categorized as a RV. The "A matrix" method is similar, but corresponds to an appropriately modified Kalman Filter (or smoother). The spdcf method typically corresponds to a full error covariance matrix of an *a priori* (unadjusted) multi-state vector, typically categorized as an SP (or RF), or as an appropriate approximation of the full error covariance matrix corresponding to a RV. Also, the spdcf method necessarily assumes that the dimension of the individual ϵX_i are the same.

The direct method has no bandwidth reduction, the "A matrix" method has appreciable bandwidth reduction, and the spdcf method has maximum bandwidth reduction. Maximum bandwidth reduction corresponds to the least amount of data necessary to faithfully assemble the corresponding full error covariance matrix.

5.8 Rigorous Error Propagation

The term "rigorous error propagation" is used to represent the proper statistical modeling of all significant errors and their interrelationships throughout an NSG system. It enables optimal solutions as well as reliable predicted accuracies associated with specific estimates and products across the system modules.

At the top-level, the statistical error model associated with a state S of a module (Collection, Value-Added Processing, Exploitation) is a necessary condition for rigorous error propagation. At a more detailed level, corresponding estimators must perform rigorous error propagation as outlined in the next section.

5.9 Estimators: WLS, Kalman Filters, etc.

Estimators, such as batch Weighted Least Squares (WLS) and sequential Kalman Filters (KF), are used throughout the NSG and have a central role regarding both accuracy and predicted accuracy. They are embedded in a subset or in all of the three main modules of a Geolocation System: Collection, Value-Added Processing, and Exploitation. Estimators and their role in the NSG are discussed in more detail in TGD 2d (Estimators and their Quality Control) and are also summarized in this section of the document:

- Section 5.9.1 – Classes and General Properties of Estimators
- Section 5.9.2 – A Representative Example: WLS in support of Multi-Image Geopositioning (MIG)
- Section 5.9.3 – Desired Estimator Characteristics: Optimality and QA
- Section 5.9.4 – Detailed Examples of QA
- Section 5.9.5 – Further Details of MIG: Sensor-Mensuration Errors

Reference [7] also provides an "easy-to-read" summary of TGD 2d.

5.9.1 Classes and General Properties of Estimators

There are a variety of classes of estimators and corresponding members within each class which an NSG application may implement, depending on requirements and operational environment. Major classes consist of batch estimators, such as the Weighted Least Squares (WLS) estimator, and (near) real-time sequential estimators, such as the Kalman Filter (KF) estimator.

WLS is typically used to adjust initial sensor metadata, including sensor pose, such as that associated with a block of overlapping images, or to later extract a feature's geolocation using (improved) sensor metadata and corresponding images. The sequential KF is typically used to estimate the sensor pose/metadata over a series of times in (near) real-time as part of a sensor platform's inertial navigation system (INS), for example, or to adjust the sensor metadata of a sequence of Motion Imagery (video) frames. It can also be used to track a moving object of interest. And because the physics related to most sensors and their relationship to measured geolocation is inherently non-linear, estimators are typically linearized about an initial *a priori* estimate of the geolocation of interest and/or the sensor metadata (trajectory).

Figure 5.9.1 -1 illustrates a subset of the more common estimators that are implemented from either the batch or the sequential class of estimators, as well as a subset of the more common properties of individual estimators from either class. A specific estimator may have more than one property.

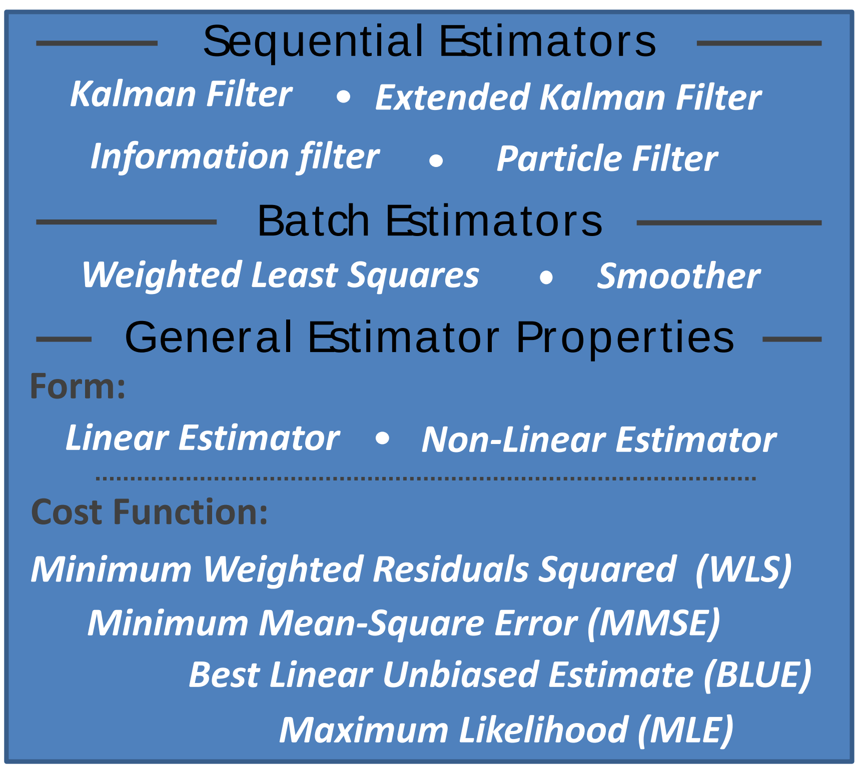


Figure 5.9.1-1: Classes of estimators and example members, and some general estimator properties

Linear and non-linear estimators correspond to the form of the estimate relative to the measurements – whether or not the solution for the state vector corresponds to a linear function of the measurements or a non-linear function of the measurements. When the state vector for solution corresponds to a non-linear function of the measurements, it is frequently linearized about a reference value or *a priori* estimate using a Taylor Series expansion, and the corresponding estimator is termed a linearized estimator.

Additional descriptors of an estimator correspond to its cost function or related properties which the estimator is designed to minimize (or maximize). An estimator is termed "optimal" if its solution minimizes its specified and legitimate cost function. For example, the cost function for a WLS solution is Minimum Weighted Residuals Squared. If the WLS solution is linearized, as is typical, the WLS solution is also BLUE, and if errors are assumed Gaussian distributed, MLE.

5.9.2 A Representative Example: WLS in support of Multi-Image Geopositioning (MIG)

In order to be both reliable and (near) optimal, estimators must perform rigorous error propagation, including: (1) linearization, (2) the input of all relevant error covariance matrices, (3) their subsequent propagation to measurement-space or state-space (typically via partial derivatives), and (4) the output of the solution's *a posteriori* error covariance matrix C_X along with solution state vector X.

The following is a representative and somewhat detailed example of a WLS estimator that includes rigorous error propagation. The example does not include processing that is in direct support of Quality Assurance (QA) which is also recommended for a real-world implementation of the estimator. However, QA for estimators in general is summarized in Sections 5.9.3 and 5.9.4 as well as detailed in TGD 2d.

The Multi-Image Geopositioning solution X is a WLS solution that corresponds to one or more 3d geolocations ("targets") measured in one or more images. If only one image is used, an external elevation source must be used as well. The solution X includes a corresponding (*a posteriori*) error covariance matrix C_X. In general, the following factors increase (improve) the solution's predicted accuracy, i.e., reduces its error covariance matrix, typically rendered graphically as a 90% confidence ellipsoid:

- Increased number of image measurements (image rays)
- Diverse imaging geometry between the collective image rays
- Increased image support data predicted accuracy (smaller support data error covariance matrix)
    - Image support data affects the image-to-ground relationship (image ray location)
    - Image support data errors are typically the dominant source of image measurement error

Figure 5.9.2-1 presents a representative example. Both the solution error and the solution 90% confidence ellipsoid are smaller for the 3-ray solution as compared to the 2-ray solution for the same geolocation, as expected. (Note: The ellipsoids appear as ellipses in the figure as it is an approximate 2d rendering of 3d geometry.)

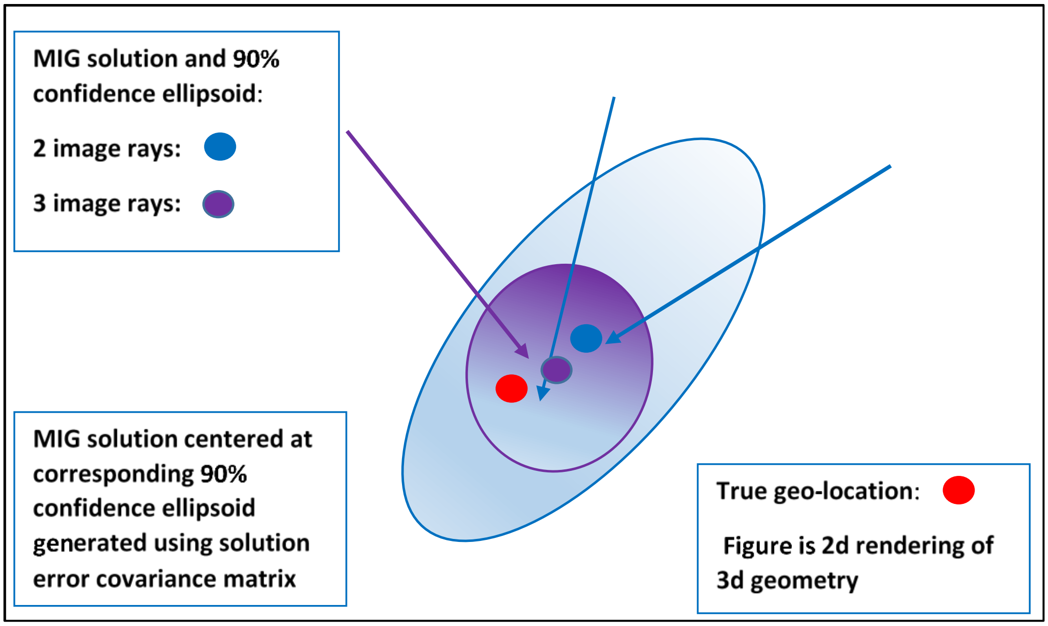


Figure 5.9.2-1: MIG Solution for one geolocation and its corresponding 90% Confidence Ellipsoid using either two or three images; the two-image solution (blue dot) is based on use of the two blue rays, the three-image solution (purple dot) is based on use of the same two blue rays and the one purple ray

In addition, the MIG solution weights the various image measurements, giving more weight to those with predicted smaller (total) measurement errors. The MIG solution X and its error covariance matrix C_X are usually computed with respect to the ECF Cartesian coordinate system, followed by the conversion of the error covariance matrix to an ENU Cartesian coordinate system. A conversion of the solution X from ECF to geodetic coordinates is also an option. Section 4.5 discusses coordinate systems and their transformation.

The equations for the MIG solution are presented below for readers interested in further detail. They illustrate rigorous error propagation via their implementation of error covariance matrices, partial derivatives, etc.

The Solution Equations

A particular Multi-Image Geopositioning (MIG) solution X is presented below, along with Table 5.9.2-1 containing corresponding solution variable and parameter definitions, including the computed *a posteriori* solution error covariance matrix C_X. This particular MIG solution is for the 3d location of two different features or ground points, based on one (line, sample) image measurement for each of these points in each of m images.

The MIG solution equations are as follows:

$$\Delta X = (C_{X0}^{-1} + B_X^T W B_X)^{-1} B_X^T W (M - M_0)$$

$$X = X_0 + \Delta$$

The corresponding table of solution variable and parameter definitions, including the all-important solution error covariance matrix C_X, is as follows:

Table 5.9.2-1: WLS MIG solution variables; m = number of images, n = number of sensor adjustable parameters per image

Variable	Variable Definition	
X	Ground location solution (3D coordinate) for two points	(6x1)
X_0	*A priori* estimate of the ground locations (X)	(6x1)
C_{X0}	Error covariance matrix of *a priori* estimate	(6x6)
M	Image point measurement (msmnts) vector	(4mx1)
M_0	Image point predicted measurement vector	(4mx1)
$C_{meas} = (B_S C_S B_S^T + \mathrm{C}_M + \mathrm{C}_{SM})$	Total measurement error covariance matrix	(4mx4m)
$W = (C_{meas})^{-1}$	Total measurement weight matrix	(4mx4m)
B_X	Partial derivatives of msmnts w.r.t the ground location	(4mx6)
B_S	Partial derivatives of msmnts w.r.t. sensor adjustable parameters	(4mxnm)
C_S	Sensor adjustable parameter error covariance matrix	(nmxnm)
C_M	Mensuration error covariance matrix	(4mx4m)
C_{SM}	Sensor-mensuration error covariance matrix	(4mx4m)
$C_X \equiv (C_{X0}^{-1} + B_X^T W B_X)^{-1}$	Solution error covariance matrix	(6x6)

Corresponding details

The $6x1$ state vector X contains the two 3d target locations. The *a priori* estimate of the targets is contained in X_0 with corresponding *a priori* error covariance matrix C_{X0}. This estimate is usually given very little weight (C_{X0} very large) unless vertical information is available from an external source (e.g., DEM) in which case X_0 and C_{X0} components are set appropriately.

The partial derivatives B_X and B_S, and the predicted image measurements M_0 are computed at the reference point X_0 using the values of the (typically) previously adjusted sensor metadata. The latter has error covariance matrix C_S with respect to sensor adjustable parameters. Note that the difference between the actual measurements M and the predicted measurements M_0, i.e., the *a priori* measurement residual $(M - M_0)$, drives the estimate of the correction ΔX to the *a priori* (reference) estimate X_0 per the above equations.

The mensuration error and sensor-mensuration error (see Section 5.9.5) are statistically represented by error covariance matrices C_M and C_{SM}, respectively. The (total) measurement error includes mensuration error, sensor-mensuration error, and the effects of sensor adjustable parameter errors, as statistically represented by the total measurement error covariance matrix C_{meas}, whose inverse is used to weight the image measurements. Note that the full sensor adjustable parameter error covariance matrix C_S is used, including the cross-covariance between sensor adjustable parameter errors between images. Furthermore, it is projected (propagated) to image space via the corresponding partial derivatives B_S prior to its addition to the total measurement error covariance matrix C_{meas}.

The use of the full sensor adjustable parameter error covariance matrix C_S is essential for an optimal solution with reliable predicted accuracy (error covariance matrix). See TGD 2d for more details regarding the content and structure of the error covariance matrices C_M, C_{SM}, and C_S.

Note that the measurement vector M has $4m$ components corresponding to a (line, sample) image measurement for each of two targets in each of m images. Also, the solution's predicted accuracy, or solution error covariance matrix C_X, improves with each additional image. Further note that the above solution equations do not include iteration for convenience and ease of notation; however, it typically is included for linearization about the (updated) operating point, along with measurement editing and the evaluation of internal performance metrics, such as *a posteriori* image residuals, for Quality Control (QC) of the specific MIG solution.

Finally, the *a posteriori* measurement residual (vector) is defined as follows and used extensively in QA-related processing as discussed in Sections 5.9.3 and 5.9.4: $\left((M - M_0) - B_X\Delta\right)$. It is essentially equal to the measurements minus their predicted value based on the solution.

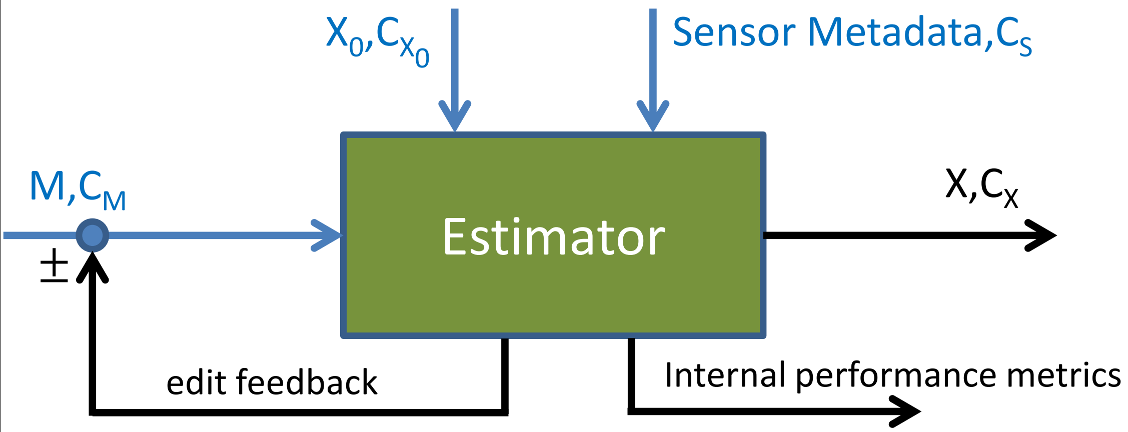
The Effect of Multiple Targets on the Solution

For ease of illustration, the above MIG solution was for two ground points or "targets". As such, the solution 6x6 error covariance matrix automatically contains the individual error covariance matrix for each point and the error cross-covariance matrix between the point pair required to compute both absolute and relative predicted accuracy. What if the solution contained only one point, or what if the solution contained many more than two points – what are the effects on accuracy and how is predicted relative accuracy computed?

In general, as the number of ground points in the solution increases, the predicted accuracy for each point gets somewhat better. When only one point is in the solution, and predicted relative accuracy is required between it and another point in a different solution which uses the same sensor support data, an appropriate formula is required. This formula is detailed, as well as the other important targeting topics in TGD 2d.

5.9.3 Desired Estimator Characteristics: Optimality and QA

A top-level summary of a generic estimator is presented in Figure 5.9.3-1 that addresses its optimality as well as its Quality Assurance.



<u>Characteristics for Optimality and Quality Assurance</u>

- Use of a priori estimates and covariance
- Appropriate weighting based on covariance
  - Measurements
  - A priori estimates
- Measurement redundancy
  - Number
  - Distribution
  - Observability
- Appropriate internal error propagation
  - Partial derivatives
  - Operating point
- Measurement Editing
  - Blunder detection
    - Measurements
    - A priori estimates
- Iteration / convergence detection / linearization if required
- Internal performance metrics
  - A posteriori residuals
  - Reference variance

Figure 5.9.3-1: Estimator and Major Characteristics for Optimality and Quality Assurance; primarily for a batch estimator – some characteristics/internal performance metrics for a sequential estimator not illustrated

Optimality is associated with achieving the best possible solution, i.e., the minimization of errors, or more specifically, the cost function. It also includes the generation of reliable predictions of solution accuracy.

Accuracy and predicted accuracy performance requirements for a system or its major modules (Section 5.1) flow down (are sub-allocated) to corresponding estimators within the modules. This, in turn, levies requirements on the information used by the estimators: the predicted accuracy, number, and distribution of measurements, as well as requirements on the predicted accuracy of *a priori* data, such as sensor metadata. These "flow-downs" also correspond to the appropriate range of operating conditions; for example, the expected range of imaging geometries when measurements correspond to images.

All of the estimator's information regarding the predicted accuracy of its inputs is assumed to be represented using members from the "tool box" of statistical error models that was discussed in Section 5.2 of this document.

The above gave a brief overview of optimality for estimators, including the flow-down of requirements to meet a desired level of predicted accuracy. Insight into what constitutes the Quality Assurance (QA) of estimators is now presented in Section 5.9.4.

5.9.4 Detailed Examples of QA

QA corresponds to the specification of applicable requirements for the implementation of an estimator, and QC corresponds to the estimator's performance of those requirements for a given solution. These requirements help to ensure that estimator solutions are reliable, including the generation of corresponding reliable error covariance matrices (predicted accuracies). QA/QC is based primarily on the internal metrics listed in Figure 5.9.3-1, but may also include occasional comparison of the solutions to ground truth, also discussed in TGD 2d.

Much of the QA/QC for batch estimators is based on *a posteriori* (post-solution) measurement residuals, while that for sequential estimators is based on *a priori* (pre-solution) measurement residuals for each update cycle (time step) of the sequential estimator. Measurement residuals are equal to the actual measurements minus their predictions based on the solution's state vector.

Two representative examples of QA/QC that are based on measurement residuals are presented in Sections 5.9.4.1 and 5.9.4.2 for those readers interested in further detail.

5.9.4.1 Measurement Editing

Individual measurement residuals normalized by their predicted covariance matrix are used for blunder detection and editing of "bad" measurements per Figure 5.9.3-1. This is applicable to both *a posteriori* measurement residuals in batch estimators (e.g. WLS) and to *a priori* measurement residuals in sequential estimators (e.g., Kalman Filter). The detection and editing of "bad" measurements help to ensure a good solution, i.e., corresponds to one form of QA/QC. The predicted covariance matrix corresponding to either individual measurement residuals or a vector of all of the measurement residuals is represented by the matrix R for convenience in this and the following section, and its computation is detailed in TGD 2d.

Figure 5.9.4.1-1 presents an example of *a priori* editing of a 2d measurement associated with a Kalman Filter, also termed a "gating test". The 2d residual corresponds to the magenta arrow which fails the test in this example because it is outside the normalized residuals' 95% probability ellipse. The latter is computed using the residuals' $2x2$ covariance matrix R that represents the summed effects of measurement errors per se and the errors in the Kalman Filter's prediction of the measurement based on its current (*a priori*) estimate of the state vector and the time of the measurement. The figure also addresses the use of two individual gating tests instead of one, where each test corresponds to only one of the two components of the 2d measurement residual and does not take into account the predicted correlation of the errors between them – the test is not as effective as the 2d test.

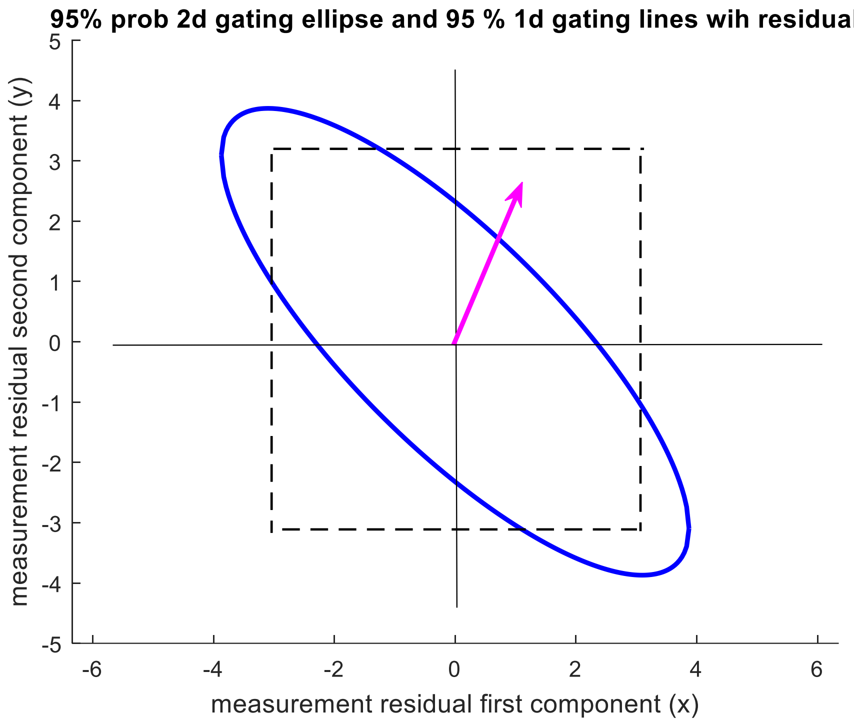


Figure 5.9.4.1-1: Ellipse corresponds to 95% confidence (probability) level for the 2d gating test and the rectangle the intersection of the dashed lines corresponding to 95% confidence 1d gating (edit) tests; the magenta line is a sample residual which will be edited by the 2d test but not the 1d tests

Similar blunder detection/editing tests to the above are also applicable to batch (WLS) estimators. However, they are typically based on all *a posteriori* measurement residuals following the solution (iteration), and are typically implemented one component at a time for efficiency. Once all tests are performed associated with the current solution, and if edits did occur, the solution is performed once again but without the edited measurements.

5.9.4.2 Reference Variance

Another form of QA/QC that is applicable to batch WLS estimators takes into account all of the *a posteriori* measurement residuals at once following the performance of any and all individual measurement residual tests. It can be used as a "final" check or test of the solution's validity. The test is based on computation of the WLS estimator's (*a posteriori*) reference variance, also listed in Figure 5.9.3-1, and defined as follows:

The WLS estimator's cost function is equal to $J = V^T WV$, which the estimator was designed to minimize. Assuming a Gaussian distribution of measurement errors, the cost function has a chi-square distribution with dof degrees of freedom. The reference variance σ_0^2 is a scalar and is equal to the post-solution value of the estimator's cost function $J = V^T W V$ divided by the degrees of freedom dof, i.e., $\sigma_0^2 = J/dof = (V^T W V)/dof$.

Regarding the computation of the cost function J, V is the column vector of *a posteriori* measurement residuals and W is the weight matrix, which is equal to the inverse of the measurement's predicted error covariance matrix R, i.e., $W = R^{-1}$. Note that, in this case, R corresponds to *a priori* measurement errors, not to *a posteriori* measurement residuals as was applicable in Section 5.9.4.1 regarding the batch estimator.

The degrees of freedom dof is essentially equal to the number of measurements minus the number of components in the state vector for solution, i.e., measurement redundancy. Also, as discussed in TGD 2d, measurements not only consist of "actual" measurements, but include any *a priori* estimates for the values of the state vector components for solution. In addition, "actual" measurement errors can correspond to the summed effects of random mensuration errors and sensor metadata errors, if applicable, such as those corresponding to *a priori* errors in sensor pose if the latter are not solved for (corrected) as part of the state vector solution.

Reference variance test

The actual reference variance test corresponds to whether the computed reference variance is within a confidence interval that is computed based on the dof and a (specified) confidence level (e.g., 90, 95, or 99%). Measurement errors are assumed approximately Gaussian distributed.

If the reference variance is within the confidence interval, the solution is determined valid; if not, the solution is invalid and a problem is indicated, typically associated with the estimator's *a priori* error models which have a major effect on the solution's *a posteriori* errors as well as on its *a posteriori* error covariance matrix or predicted accuracy.

The expected value of the reference variance is equal to 1, and both its expected dispersion about its mean value and the length of confidence intervals are greater the smaller the dof.

A new and recommended reference variance test

The reference variance tests that are described in TGD 2d include both the "standard" reference variance test that was described above as well as a new, more robust, and recommended reference variance test that takes into account the fidelity of the measurement error covariance matrix R, which is never perfect. More specifically, fidelity is characterized as:

$$\max(0, (1 - ci\_tol))R < R_{true} \leq (1 + ci\_tol)R,$$

where R_{true} is the true but unknown value for R and ci_tol a specifiable tolerance for their relative deviation. This is illustrated in Figure 5.9.4.2-1 using corresponding error ellipses and a value of $ci\_tol = 0.2$. R in the figure corresponds to the covariance matrix for the errors in one of the $2x1$ measurements that are within the collection of $2x1$ measurements that make-up the entire measurement vector. Note that the ellipses in the figure associated with error covariance matrices $1.2R$ and $0.8R$ are $\sqrt{1.2}$ and $\sqrt{0.8}$, respectively, times the size of the ellipse associated with the error covariance matrix R. That is, they are about 10% larger and smaller, respectively.

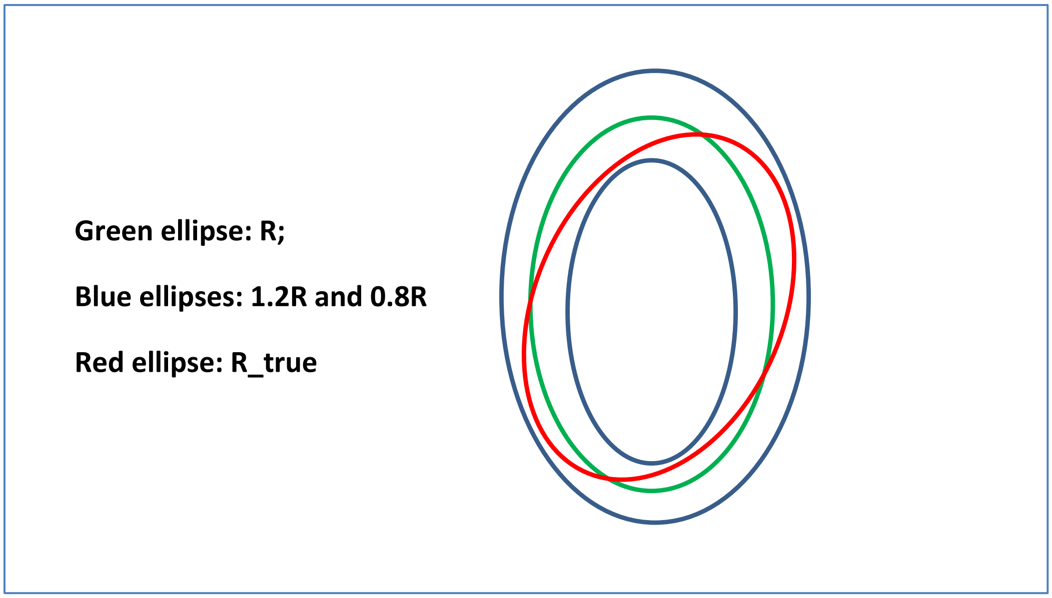


Figure 5.9.4.2-1: Relationship between R and R_{true} based on error ellipses; probability of error ellipses arbitrary as long as common; relative scale of error ellipses exaggerated somewhat for clarity

Note that the orientation of the green ellipse associated with R is actually solution-specific and that its scaled versions (blue ellipses) are oriented in the same way by definition. The red ellipse associated with R_{true} can be anywhere between the blue ellipses when the specified fidelity is satisfied. Also, the formal definition for a covariance matrix A less than or equal to a covariance matrix B of the same dimension, i.e., $A \leq B$, is that the matrix $(B - A)$ is positive semi-definite, i.e., all of its eigenvalues are non-negative. See Section 5.5.3.1 for further background regarding error covariance matrix inequalities and TGD 2a (Predictive Statistics) and TGD 2d for further details regarding the above application.

The test on the validity of the estimator's solution is based on a confidence interval test for the reference variance that also takes into account the specified tolerance (0.2 in the above example). The test involves the length of the confidence interval bounding the reference variance's expected value of 1, and takes into account the probability level of the confidence interval (e.g., 90, 95, or 99%), the degrees of freedom of the estimator's solution, and the above tolerance associated with the value of R that is used in the WLS solution. For example, if the computed reference variance is within the (new) 95% confidence interval, we are 95% sure that the solution is valid relative to the specified tolerance or level of fidelity of the measurement error covariance matrix R.

The use of the new reference variance test enables realistic and useful tests regarding the validity of the estimator's solution. If the standard or classic reference variance test were used instead, the test almost always fails in the presence of a large degree of freedom (many redundant measurements), typically associated with large block adjustments of sensor metadata. This is because the confidence interval that is based solely on R becomes unrealistically small in the presence of many measurement, which is almost always the case since error modelling is never perfect.

The use of the new reference test also makes the scaling of the estimator solution's error covariance matrix by the reference variance unnecessary. Scaling by the reference variance is sometimes done in practice in order to enforce a more realistic value of the solution's error covariance matrix (predicted accuracy) in the presence of mis-modelling, but it is not recommended per reasons discussed in TGD 2d.

If validity checks fail too frequently, this indicates that there is a problem with modeling associated with the estimator. It should be corrected based on the available QA/QC metrics and a review of the estimator's design details, *a priori* error models, and should also include a review of the number and the definition of the components in the state vector for solution.

5.9.4.3 QC actions associated with a specific solution

Computation and analysis of the various Quality Assurance metrics in Figure 5.9.3-1 for an estimator's specific solution corresponds to Quality Control (QC) of the solution. Results indicate whether the solution is valid or not.

In addition, detailed analysis of corresponding metrics over multiple solutions supports improvements in overall estimator modeling and/or design, if applicable, and which is typically limited to improvements ("tuning") of the *a priori* error models, e.g., modified values for various *a priori* error covariance matrices.

This is discussed in further detail in TGD 2d. In addition, and of great benefit in the making of any such improvements, is "calibration", as described at the top-level in Section 5.10.

5.9.5 Further Details of MIG Error Propagation: Sensor-Mensuration Errors

This section of the document discusses a relatively new concept associated with rigorous error propagation: sensor-mensuration error and its representation using predictive statistics. It was referenced briefly in Section 5.9.2 regarding the MIG WLS solution. It is further detailed below for the interested reader.

As typical for an estimator, there were multiple sources of error addressed in the MIG WLS solution: errors in the *a pri*ori estimate of the state vector for solution, errors in the sensor metadata affecting the predicted measurements, mensuration errors in the explicit measurement process itself, and errors which are termed "sensor-mensuration errors" (which have been referred to in the past as "unmodeled errors"). The latter source of error is a somewhat recent concept, but vital to reliable solution predicted accuracy, in particular, for reliable predicted relative accuracy when more than one ground point location is solved for simultaneously, as is frequently the case for a "mensuration application" (e.g., extraction of a linear feature such as a runway).

For multiple ground points solved for together in MIG, and particularly for monoscopic MIG (one image with elevation source), the effects of sensor metadata errors are very similar for points close together; hence, tend to cancel out with negligible effect on relative accuracy between ground point pairs. Mensuration errors are uncorrelated and do not cancel out, but have the same statistical effect independent of how close together the points are. Sensor-mensuration errors, on the other hand, have a statistical effect that typically grows with distance between points – a typical observed effect in experimentation/testing of commercial satellite imagery.

Sensor-mensuration errors correspond to the effects of sensor errors that are too "high-frequency" to be represented as errors in explicit sensor metadata adjustable parameters, such as 3d position correction for image i, $i = 1,..,m$. Therefore, their effects on image measurements are statistically represented directly in image (measurement) space. A separate (line, sample) error corresponds to each image measurement for all images involved. These errors are uncorrelated across images, but correlated within an image. They are modeled as corresponding to a 2D RF (2d) for each image as follows.

A $2x2$ error covariance matrix and the degree of spatial correlation for an image are specifiable, and correspond to the 2D RF (2d), where 2D "space" is (line, sample) image-space directions and (2d) components are line error and sample error. A typical strictly positive definite correlation function (spdcf) used to represent the "spatial" correlation is presented in Figure 5.9.5-1.

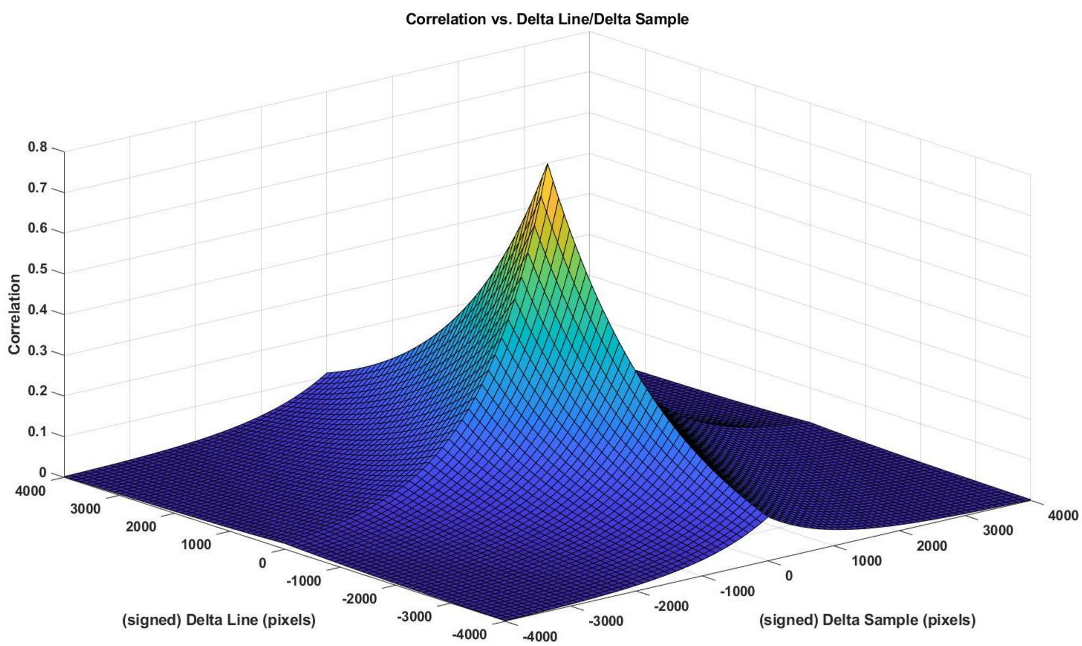


Figure 5.9.5-1: Sensor-mensuration error: spatial correlation of the corresponding 2D RF (2d)

Note that different spatial correlation can be assigned to different directions in the image (anisotropic). Also, the maximum correlation (coefficient) is less than 1.0 at negligible distance and decreases with increasing distance between a point pair. Thus, the effect of sensor-mensuration error on relative accuracy is always non-zero, increases with distance, and then levels-off as correlation approaches zero to a maximum value dictated by the corresponding error covariance matrix for the 2D RF (2d).

This is illustrated as follows for the line-component of sensor-mensuration error only for simplicity, i.e., only the variance $\sigma^2_{line\ \epsilon}$ for line error contained in the $2x2$ error covariance matrix is applicable, along with the line error's spdcf represented as $\rho_{line\ \epsilon}(\Delta line, \Delta sample)$, where $\Delta line$ and $\Delta sample$ are the line and sample distances between the two points of interest in the image:

$rel\sigma^2_{line\ \epsilon} = 2\sigma^2_{line\ \epsilon}(1 - \rho_{line\ \epsilon}(\Delta line, \Delta sample))$,
the variance of relative line error between the two points.

The variance is small for two points close together and increases as the distance between them increases. The above equation for the variance represents the predicted relative accuracy in image-space (line component only) due to sensor-mensuration error. Both the corresponding error covariance matrix and the spatial correlation function are predictive statistics corresponding to the 2D RF (2d) representing sensor-mensuration error for the image(s) and their support data.

Sensor-mensuration error is further detailed in [9] and the corresponding error modeling in TGD 2d. Further note that sensor-mensuration error is known as "unmodeled error" in [10] and other previous documentation, a misnomer that we are trying to correct. It is a misnomer because, although not modeled functionally such that it is adjustable or correctable, it is modeled statistically.

5.10 Accuracy and Statistical Error Model Periodic Calibration

Rigorous error propagation in general, and optimal estimators in particular, require a reasonable statistical error model(s) for the corresponding NSG module(s). It is unrealistic that such models are always available, either at system "start" or throughout operations. Their general form may be reasonable, but parameter values describing their specifics may not be. Thus, the statistical error models corresponding to predicted accuracy must be "calibrated" periodically. This typically requires that the simpler, probabilistic-based error model corresponding to (system) accuracy be calibrated first, with results "flowing down" to the statistical error model(s) associated with predicted accuracy.

An important input to the above process is the periodic assessment of accuracy and predicted accuracy, essentially the data and procedure of Validation discussed earlier in Section 5.1, but done for calibration purposes, not system validation, and done over a "calibration range", such as multiple fields of surveyed ground control points. This process also serves as an important part of general system accuracy Quality Control.

5.11 Monte-Carlo Simulation of Errors for Simple and Complex Systems

Section 5.8 of this document discussed rigorous analytic error propagation. However, it can be difficult to perform rigorous error propagation analytically for a complicated system due to non-linear effects, and impossible if it is a "black box".

Monte-Carlo (sample-based) simulation can be used instead as outlined in Figure 5.11-1 and as detailed in TGD 2e (Monte-Carlo Simulation), including corresponding pseudo-code. With the appropriate approach, throughput is typically no longer an issue; although there still remain trade-offs regarding generality versus speed. Approaches detailed in TGD 2e include the ability to simulate correlated input samples in a very fast manner.

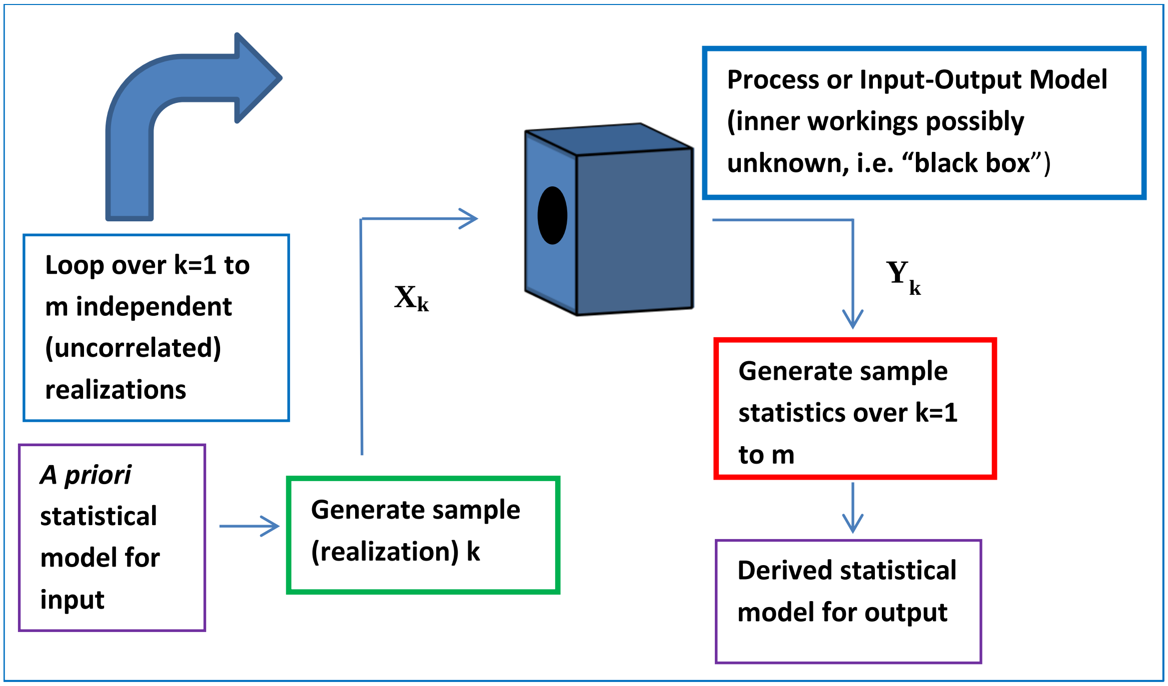


Figure 5.11-1: MC Simulation Overview

Note that sample (realizations) are generated for system inputs based on an assumed *a priori* (predictive) statistical model, and then sample statistics are generated over the corresponding system output samples and a (sample) statistical model derived. The *a priori* statistical model for system inputs can correspond to random variables, random vectors, stochastic processes, or random fields that typically represent errors. The random variables and random vectors are sometimes referred to as "stand-alone" random vectors in order to differentiate them from a collection of random vectors associated with a stochastic process or a random field.

5.11.1 Simple examples corresponding to the simulation of random vectors

This section presents simple examples of the generation of independent samples or realizations of (stand-alone) random vectors and independent samples or realizations of random variables, respectively. The first example assumes a multi-variate Gaussian distribution for the random vectors and the second example assumes an arbitrary and specifiable probability distribution for the random variables, which can also be extended to random vectors assuming uncorrelated components. Although the underlying algorithms are relatively simple, particularly for the first example, they are very effective and useful for many applications.

A simple example: generation of independent samples of random vectors with a Gaussian probability distribution

A simple example of the generation of input realizations, the left side of Figure 5.11-1, corresponds to the sequential generation of independent samples or realizations of an $nx1$ random vector X_k that is consistent with a (multi-variant) Gaussian distribution and a specified (*a priori*) $nx1$ mean-value $\bar{X}$ and nxn valid (symmetric and positive definite) covariance matrix C_X :

$$X_k = C_X^{1/2} r + \bar{X},$$

where $C_X^{1/2}$ is defined as either the principal matrix square-root of C_X or the Cholesky decomposition of C_X, and r is an $nx1$ vector of n independent realizations of a Gaussian or Normal $N(0,1)$ random variable, i.e., Gaussian distributed with a mean-value of zero and variance of one. Both of the matrix square-root functions as well as the generation of independent realizations of $N(0,1)$ random variables are readily available in the functional libraries of most programming languages, such as in MATLAB. Because the covariance matrix C_X can be specified as non-diagonal, components in each random vector can be specified as correlated (intra-state vector correlation).

The above need not necessarily support the generation of inputs for the general Monte-Carlo paradigm indicated in Figure 5.11-1 that may involve a somewhat complicated Process Module, but can correspond to a Process Module that is "pass-through" as well, i.e., the simulation of a collection of independent samples of known statistical characteristics is of direct interest per se. This was the case in support of many of the examples in the TGD level-2 documents involving simulated errors.

<u>Simultaneous generation of multiple samples</u>

TGD 2e also details a similar but faster method to generate multiple independent samples than to generate them sequentially one at a time as was detailed above. The computation of 1,000,000 2d samples on a lap-top computer was done in approximately 0.1 seconds using provided pseudo-code.

In particular, a $2x1$ random vector was of interest corresponding to horizontal x-component and y-component geolocation errors. Its non-zero *a priori* mean-value and its error covariance about this mean-value were specified as follows:

$\bar{X}^T = [10 \quad 5]$ meters and $C_X = \begin{bmatrix} 10^2 & 0.75 \times 10 \times 12 \\ . & 12^2 \end{bmatrix} = \begin{bmatrix} 100 & 90 \\ 90 & 144 \end{bmatrix}$ meters-squared.

10,000 of the simulated 1,000, 000 samples are illustrated in Figure 5.11.1-1, which includes the corresponding scalar accuracy metrics CE50 and CE95 computed using the samples based on order statistics as discussed in both TGD 2e and TGD 2a (Predictive Statistics).

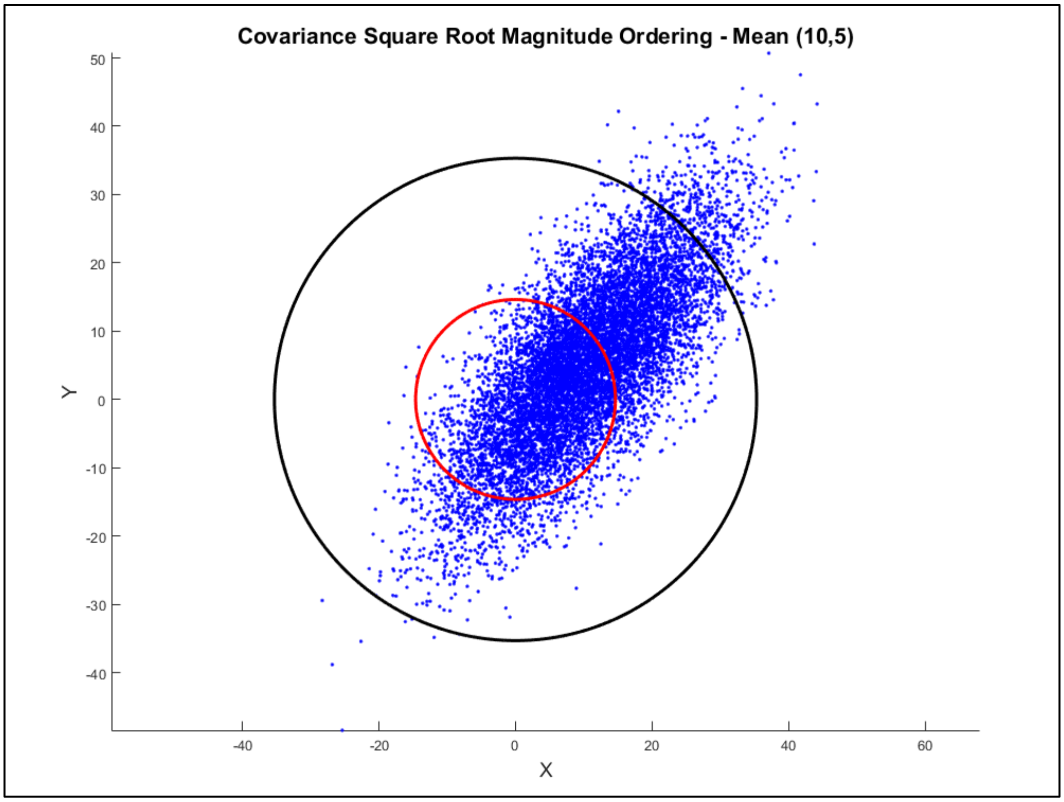


Figure 5.11.1-1: 10,000 of the 1,000,000 random samples (blue dots); includes the CE50 circle (red) and the CE95 circle (black), where both circles are centered about the origin (zero) instead of the mean-value by definition of CE; on the other hand, the sample mean-value closely approximates the *a priori* mean-value of x=10 and y=5

A simple example: generation of independent samples of random variables with an arbitrary probability distribution

TGD 2e also details how to generate independent samples or realizations of random variables with an arbitrary but specifiable probability distribution. It can also be used to generate independent samples of random vectors, but unlike the multi-variate Gaussian distributed example above, it requires the added restriction that the components (random variables) in each random vector are uncorrelated.

The general technique is termed "inverse transform sampling". And although this technique is not summarized further in this document, Figure 5.11.1-1 presents a representative example of its implementation. The red line corresponds to a "custom" specified probability density function (pdf) for a random variable, and the blue bar graph corresponds to the histogram of 20000 independent samples generated based on this technique. For ancillary information, the corresponding mean-value and standard deviation of the random variable x were computed from the specified $pdf(x)$ and are equal to approximately 0.92 and 3.4 meters, respectively.

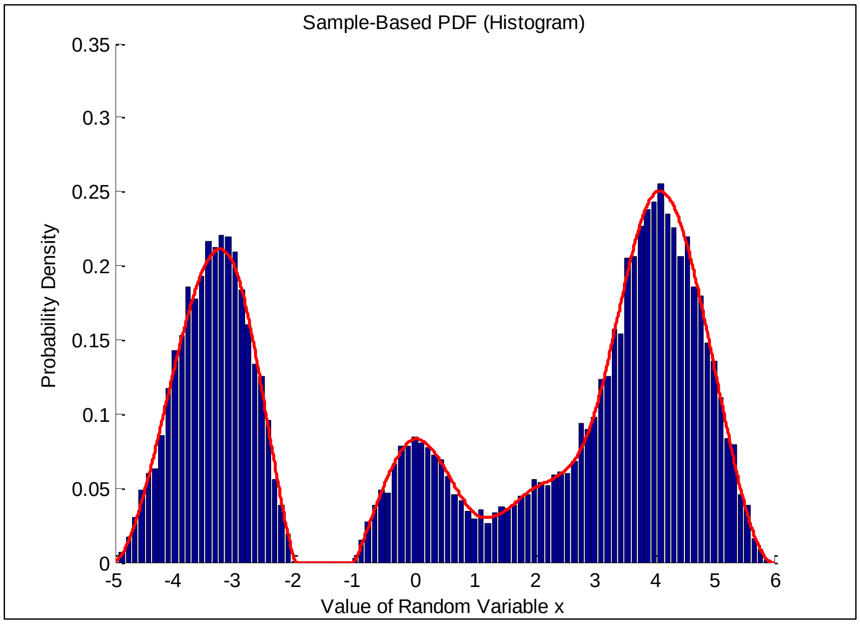


Figure 5.11.1-1: Specified custom analytic pdf (red) and corresponding sample-based pdf (histogram, blue) of 20,000 independent samples or realizations

The above pdf is significantly different than a Gaussian pdf. A Gaussian $pdf(x)$ over roughly the same domain is illustrated in Figure 5.11.1-2 below for qualitative comparison, along with a histogram of 20000 independent samples. The Gaussian distribution has a mean-value of 1 and a standard deviation about the mean of 3 that completely characterize the pdf and which corresponds to the familiar bell-shaped curve.

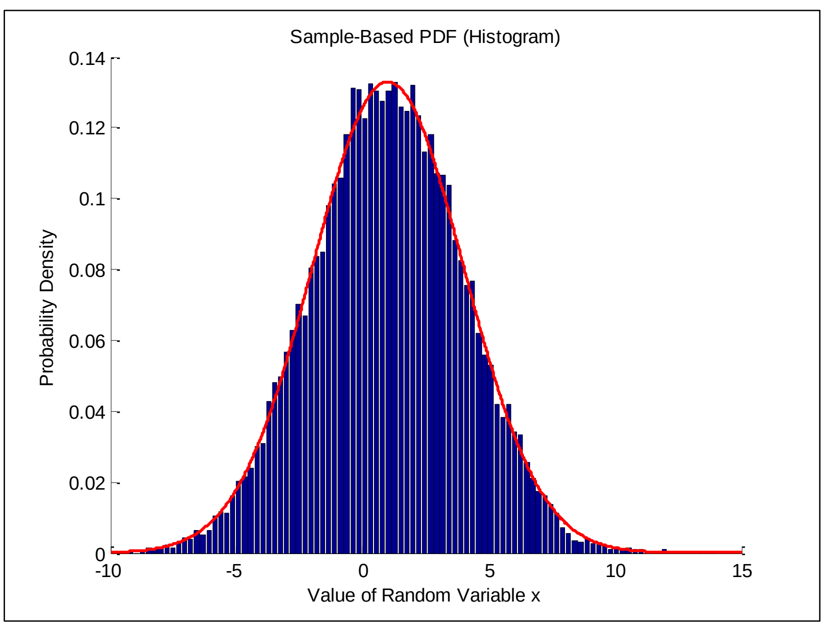


Figure 5.11.1-2: Gaussian pdf (red) and 20,000 independents samples or realizations

5.11.2 General examples of Monte-Carlo simulation embedded in applications

Other applications of the sequential generation of independent samples with known *a priori* characteristics not only involve the more general Monte-Carlo paradigm, as summarized in Figure 5.11-1 and as detailed in TGD 2e, but the corresponding Process Module is actually embedded in specific lower-level operational applications. Two such examples are outlined below: Viewshed and Non-linear MIG. The former simulates errors represented as random fields, and the latter simulates errors represented as both random fields and as (stand-alone) random vectors. All realizations of error were also simulated consistent with a Gaussian probability distribution.

As discussed earlier in this document, a random field consists of a collection of spatially correlated random vectors (inter-state vector correlation). On the other hand, (stand-alone) random vectors are uncorrelated but their components or random variables may be correlated within the same random vector (intra-state vector correlation).

The simulation of the random fields was performed using the Fast Sequential Simulation (FSS) algorithm as documented in TDG 2e. The algorithm is very fast. For example, it can simulate a scalar homogeneous 2D random field across a horizontal grid of 20,000,000 locations in approximately 1 second using a laptop PC. The corresponding random field is symbolized as 2D RF (1d) per the definitions presented in Section 5.3. The term 1d corresponds to scalar (error) and the term 2D corresponds to horizontal grid.

The algorithm assumes a Gaussian distribution of errors and spdcf that correspond to exponential decay as a function of distance, the latter a common and practical form for spatial correlation. Figure 5.11.2-1 presents an overview of the generation process for a simulated realization of the random field, typically for spatially correlated errors (z) across a grid (l, k) aligned with horizontal (x, y) space.

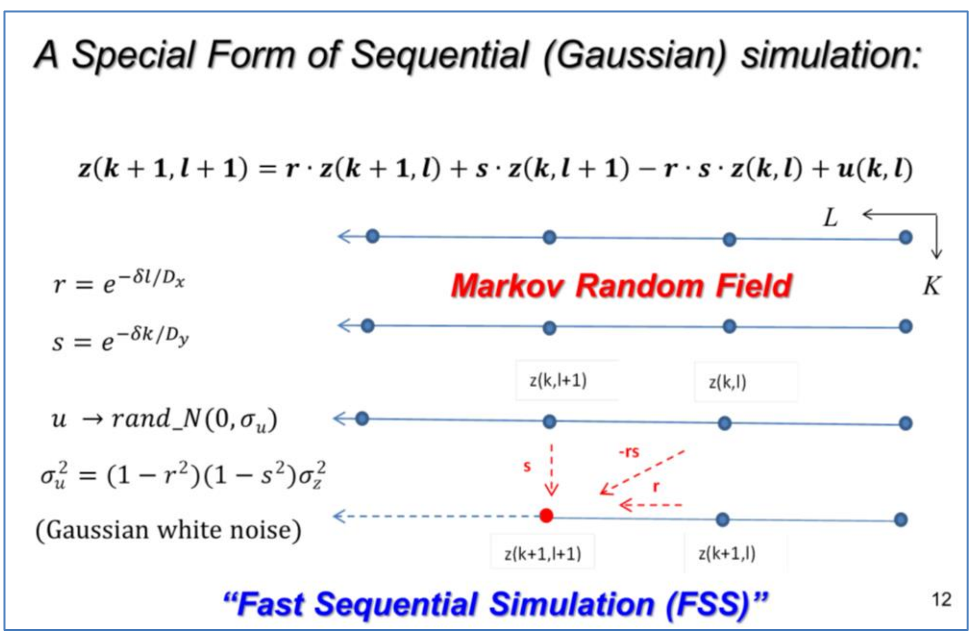


Figure 11.2-1: FSS sequential generation of a realization of a scalar homogeneous 2D random field

Pseudo-code for the algorithm is also included in TGD 2e that simulates 1D, 2D, 3D, or 4D random fields, the latter typically corresponds to a 4D grid aligned with (x, y, z, and time) coordinates. The document also describes the simulation of multi-variate errors ($nd, n > 1$) as well as non-homogeneous random fields.

Viewshed Example

Viewshed is a common spatial analysis technique by which to assess what is visible by an observer (or conversely the observer's visibility) for a given location.

Figure 5.11.2-1 presents a portion of the ocean's bottom surface (aka ocean floor) represented as a horizontal grid of depth generated from bathymetric 3d survey data, as detailed in TGD 2e. The grid contains *N*=300,000 points, and the distance between grid points is 1000 meters in each horizontal (X and Y) direction. An observer's location of interest is represented by the light blue circle 50 meters above the bottom surface.

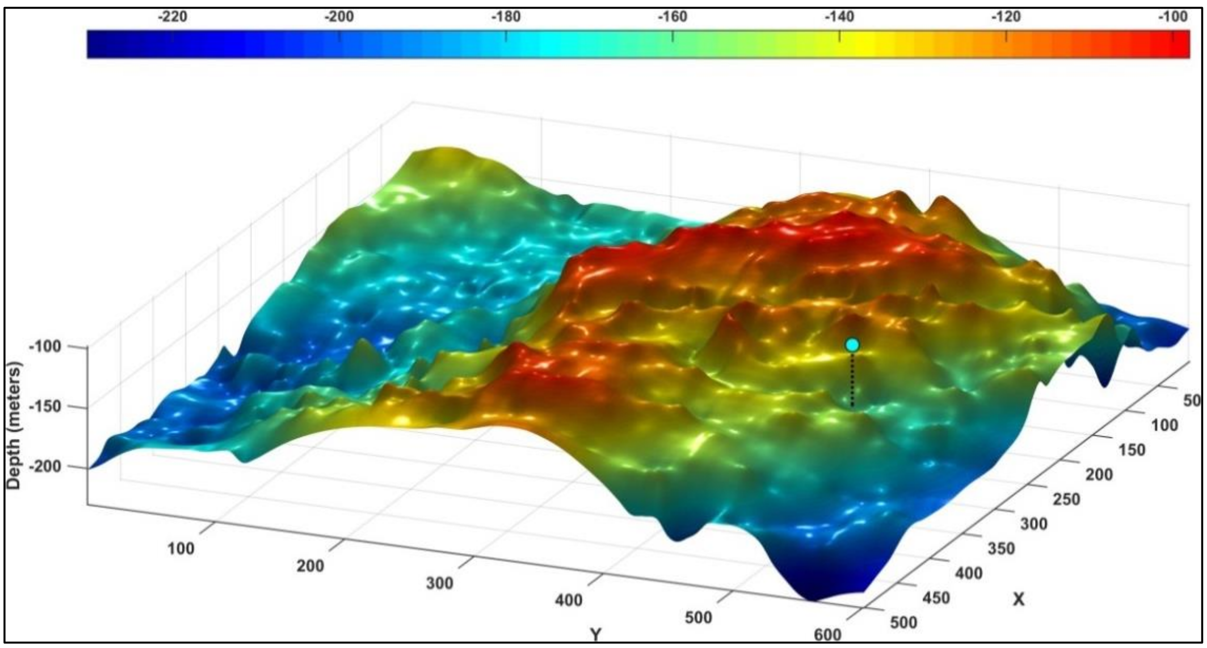


Figure 5.11.2-1: Horizontal grid of ocean bottom surface (color bar corresponds to depth in meters)

The actual bottom surface is also assumed to have an additional surface z (scalar) error represented by a non-homogeneous 3D RF (1d). The standard deviations for z error is 2.5% of depth. The spatial correlation is either modeled as spatially uncorrelated across all three (x,y,z) directions, or spatially correlated, modeled as a separable (product of three) decaying exponential with distance constants of 5000, 5000, and 250 meters, respectively.

Figures 5.11.2-2 and 5.11.2-3 show probabilistic viewshed outputs draped over the bottom surface of Figure 5.11.2-1, for spatially uncorrelated and spatially correlated errors, respectively. The viewshed probabilistic outputs were generated by summing the results of 100 independent viewshed outputs. Each viewshed output is a binary visibility/invisibility value for each of the *N* grid points or cells. A specific viewshed output was generated by the viewshed algorithm, given the location of the observer and an independent representation of the bottom surface over the *N* cells. The representation of the bottom surface corresponds to the original bottom surface (reference) plus the trilinear interpolated results of an independent realization of the scalar random field. The scalar corresponds to vertical or depth error (*z*) over a 3D (*xyz*) grid that contains the original bottom surface and its grid within.

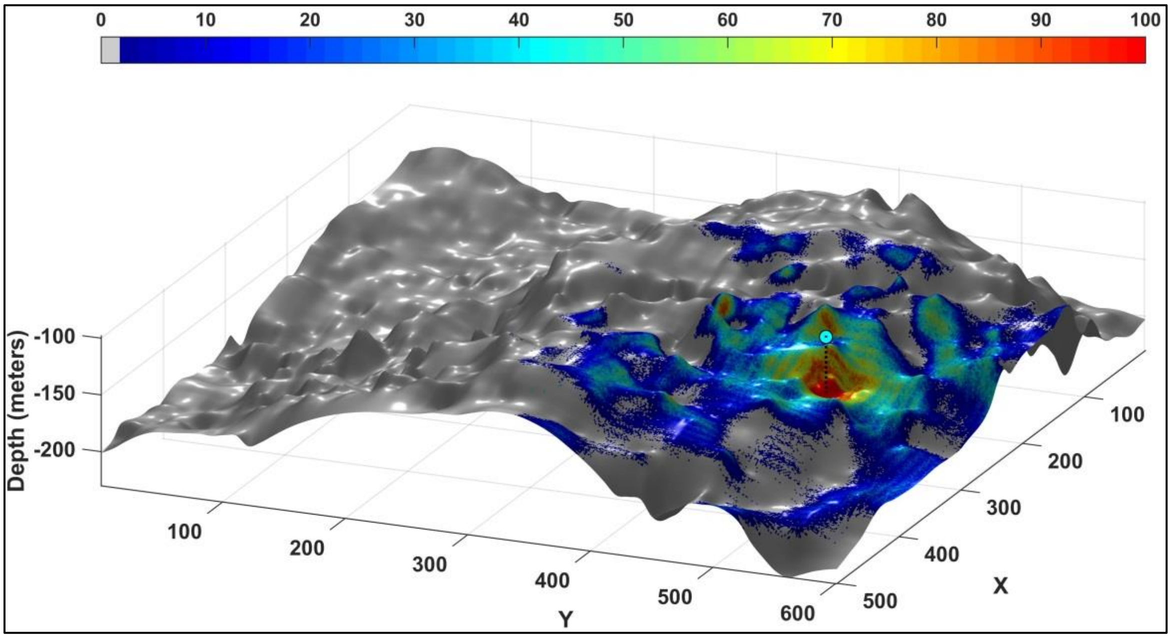


Figure 5.11.2-2: Probabilistic viewshed from uncorrelated z error (color bar represents the probability that the bottom surface is visible to the observer)

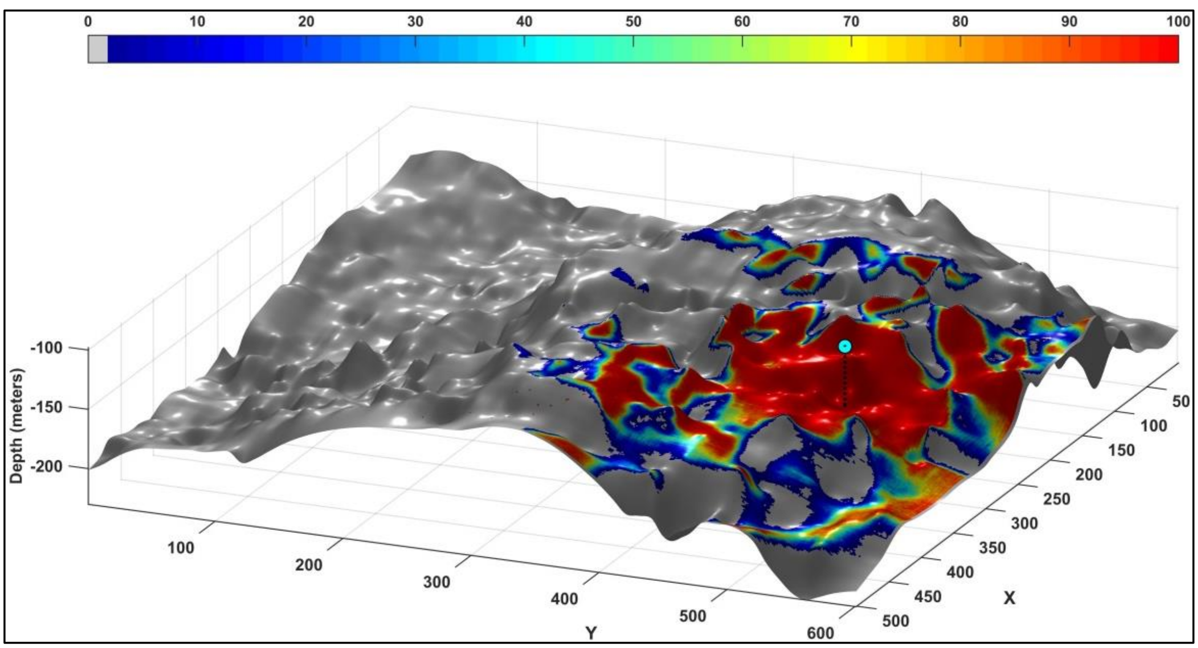


Figure 5.11.2-3: Probabilistic viewshed from spatially correlated z error (color bar represents the probability that the bottom surface is visible to the observer)

The viewshed output for the spatially correlated case contrasts dramatically with the uncorrelated case by showing significantly higher probabilistic visibilities. See reference [11] for more details regarding the above viewshed example.

Non-linear MIG Example

Monte-Carlo simulation can also be embedded, possibly as an option, in straight-forward exploitation applications when appropriate. For example, consider the case of monoscopic MIG (aka SIG) which corresponds to image-to-ground at a specified elevation, i.e., the intersection of the line-of-sight or image ray with a DEM or DSM. However, for this particular application, the nominal 3d ground point position or "operating" point for the solution is "unstable", i.e., linearization about the solution is problematic. This is depicted in Figure 5.11.2-4. The 3d "ground point" is near the corner of the building's roof top.

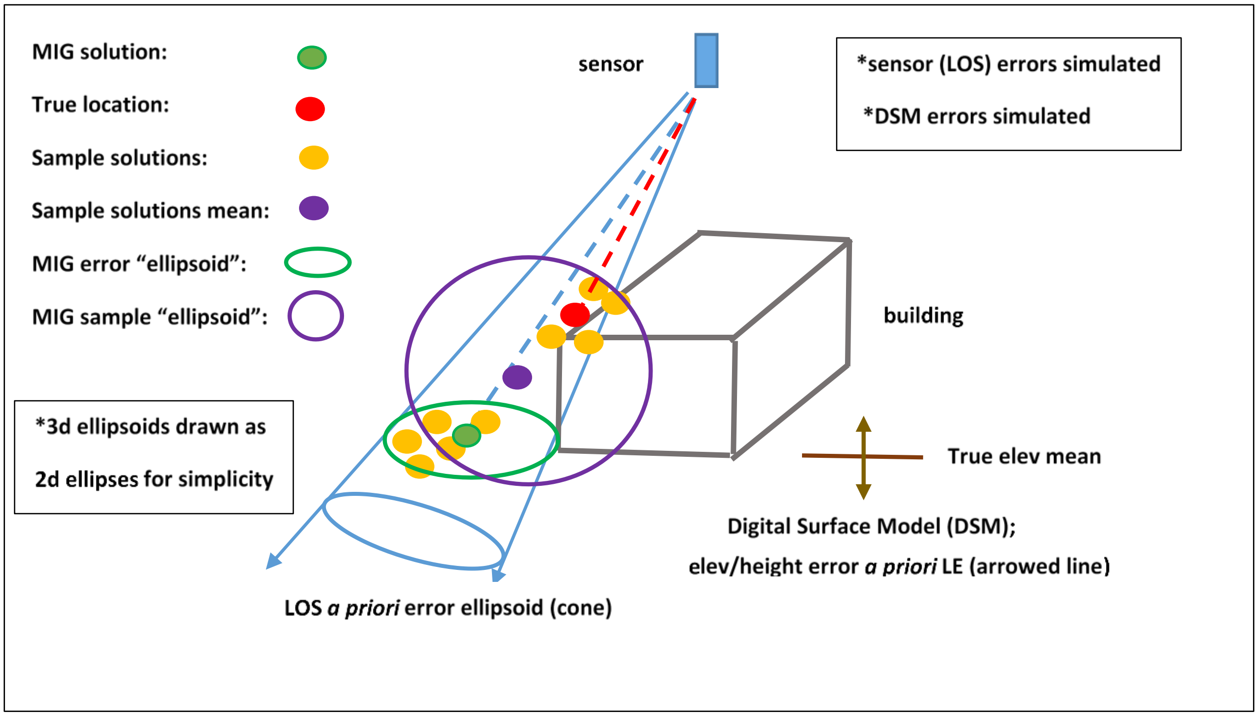


Figure 5.11.2-4: MIG mono extraction based on an EO sensor: analytic versus simulation-based solution at unstable operating point; operationally, there are more simulated sample solutions (gold dots) than are depicted in the figure

There are two MIG solutions, the nominal MIG analytic solution (green dot), and the MIG simulation-based solution (purple dot). The samples correspond to simulated independent realizations of sensor support data (line-of-sight) error, as well as independent realizations of Digital Surface Model (DSM) error, with corresponding MIG sample solutions (gold dots). The MIG simulation-based solution corresponds to the mean-value of the MIG sample solutions. Monte-Carlo simulation of errors (realizations) are based on the corresponding error models or error covariance matrices as detailed in TGD 2e (Monte-Carlo simulation). More specifically, DSM errors are simulated as realizations of a random field, and sensor support errors are simulated as realizations of a random vector. For each realization of the random field, multiple realization of the random vector are performed.

The specific extraction scenario is as follows. A pixel in the image corresponding to a location on the building roof-top near a corner is identified and measured in the image. The available image sensor support data is incorrect, as expected, and as represented by its nominal LOS (blue dashed line) corresponding to the pixel location and its corresponding error ellipsoid (blue ellipsoid or cone) centered about the LOS due primarily to sensor metadata errors or predicted accuracy.

The MIG analytic solution (green dot) corresponds to the nominal and incorrect LOS intersected with the DSM, and the surrounding green ellipsoid represents the solution's predicted accuracy. The analytic solution intersects the ground, not the roof top. The true location (red point) corresponds to the correct LOS (red dashed line) and the roof top's true DSM value. The MIG simulation-based solution is the purple dot with surrounding 90% sample error ellipsoid. The disparity between the MIG analytic solution and the MIG simulation-based solution indicates a problem. Once identified, the problem can be mitigated. For example, the MIG solution is constrained to use an approximate elevation/height corresponding to the building rooftop.

5.12 External Data and Quality Assessment

The NSG is becoming increasingly more reliant on External Data: ranging from "semi-external" outsourcing of tasks, to external commodity and crowd-sourcing data. For the latter two, assessing accuracy and quality (reliability) is and will continue to be challenging, as detailed metadata and pedigree may be nil. Outsourced data is usually generated against an NSG-supplied specification of performance requirements. The challenge is to continuously ensure, as best as possible, that the product requirements are being met without formal and expensive (re)testing. Figure 5.12-1 is a graphical depiction of the overall process of accuracy and quality assessment of External Data in the NSG.

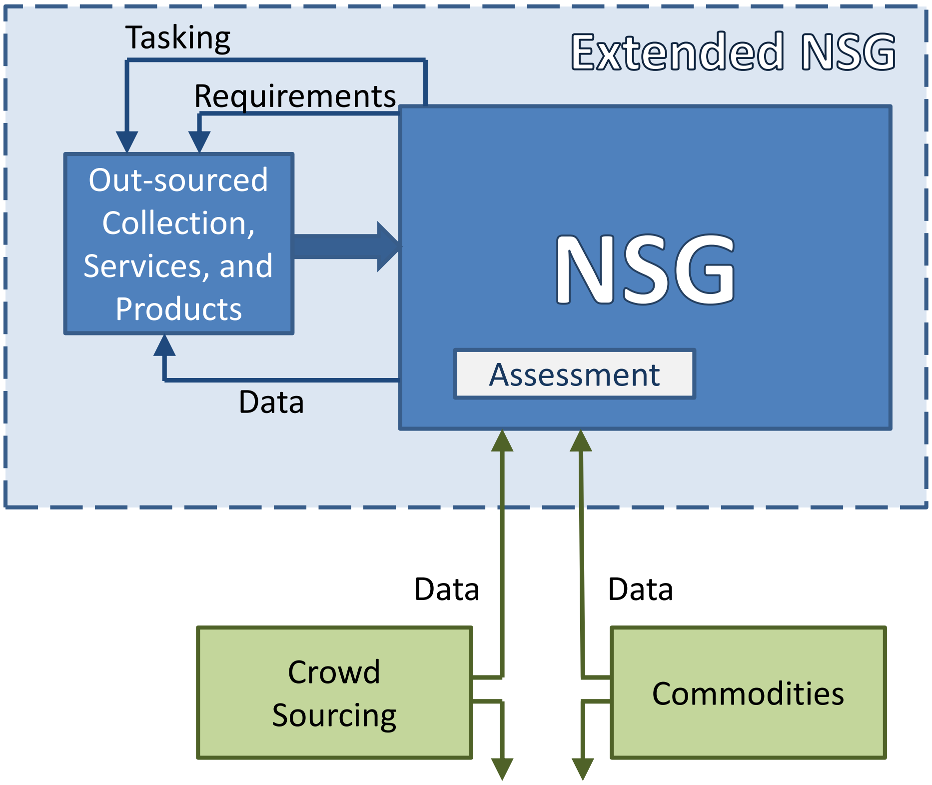


Figure 5.12-1: Functional flow of External Data into the NSG

This general subject is covered in TGD 2f (External Data and its Quality Assessment) and briefly discussed here:

For outsourcing, some Quality Assurance (QA), as opposed to simply quality assessment, of the outsourced product is typically built-in to the requirements for the particular outsourcing contract. However, the "tasking" module within the NSG would like more confidence regarding the Quality Assurance (QA) and corresponding Quality Control (QC) for each specific product delivered without the expense and delay of detailed testing on a per product-delivery basis. One approach is to include in the requirements that the data for internal QC checks be delivered by the contractor along with the nominal product, so as to ensure that these checks were indeed performed by the contractor (or at least the required internal metrics were generated). In addition, the NSG tasking module can review these results with appropriate feedback to the contractor, if necessary.

Of course, the specific QA/QC internal metrics vary with the type of outsourced product. As an example, for outsourced image registration task involving a large number of overlapping images (aka "triangulation" or "bundle adjustment"), internal metrics could include detailed shear statistics (not just a one or two number summary per model), detailed y-parallax statistics, number and distribution of tie points used, and various (WLS batch) estimator internal performance metrics, such as the measurement residual Chi-Square value, values of the various parameter corrections normalized by their *a priori* error covariance, internal measurement editing results, etc. These types of metrics can ensure that the solution was at least internally consistent.

The estimation of accuracy and the quality assessment of externally generated data, such as commodities and crowd-sourcing data, is more difficult, as the NSG has virtually no control of the data generation and its internal QA (if done at all) process. In addition, the range of data is virtually unlimited: (1) Small-Sat imagery with little metadata, (2) various feature data bases, and (3) collections of independent photographs over an object of interest, for example. Recommended NSG approaches and methodologies include the following:

- Generation and update of NSG-internal quality assessments of External Data
    - Quality assessments based on the type of External Data: commodities or crowd-sourcing data
    - Further sub-categorized by vendor or collector, applicable date-range, etc.
- Assessment of crowd-sourcing data including comparisons to similar data between multiple collectors
    - Example: openStreetMap, Wikimapia, Google Maps, etc.
    - Overall quality possibly a function of the number of "people" generating the crowd-sourced data of interest, the time-interval, and the base-layer
    - Typically, a smaller amount of quality-related information is available for crowd-sourcing data than is available for commodities data, and that which is available, typically less reliable
- Assessment of commodities data including the population of models which are then stored, periodically updated, and made available to general users of the data for (near) optimal and informed use
    - Accuracy assessment models include sample statistics of error relative to ground-truth or its equivalent
        - Selection of which particular statistics are included in the models taking into consideration the sparse number of samples that are typically available
    - Predicted accuracy models include predictive statistics (mean-value, covariance matrix, spdcf) that are typically "tuned" using the corresponding accuracy assessment model
        - Allows users to properly utilize commodities data (past, present, future) associated with the same sub-category as the predicted accuracy model by taking geospatial uncertainty into account
- Maintenance of an over-all NSG database, preferably by a designated NSG government organization, by the compilation and categorization of the results of the above processing, performed periodically by both the designated organization as well as other NSG organizations

The general task of accuracy prediction and quality assessment of External Data used by the NSG is a current and future (on-going) research area, with more details presented below that represent a subset of the current recommendations that are further detailed in TGD 2f.

5.12.1 Representative Examples of External Data: Commodities and Crowd-Sourcing Data

Prior to presenting more details regarding current recommendations, we present representative examples of commodities and crowd-sourcing data.

Commodities Example: Small-Sat Imagery

Figures 5.12.1-1 and 5.12.1-2 present an overview of Small-Sat imaging satellites and their corresponding images, respectively.

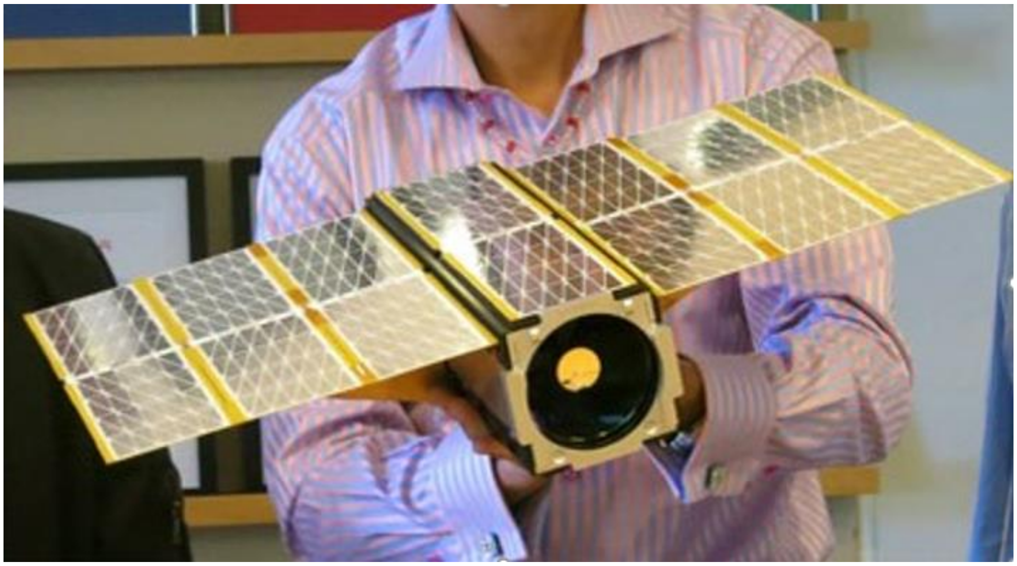


Figure 5.12.1-1: One of approximately 200 Planet Dove Small-Sats; 3-5 m ground-sample distance [2]; additional permission to use via "Source:@Year, Planet Labs Inc, Contract HM0476-18-C-0044"

Figure 5.12.1-2: Planet Dove Image of El-Alamein Egypt, Aug 28. 2016; from Planet Lab's web-site [16]; permission to use via "Source:@Year, Planet Labs Inc, Contract HM0476-18-C-0044"

In general, Small-Sat images and their corresponding metadata are becoming more and more prevalent and directly applicable to various geolocation applications. However, although their metadata supports implementation of the image-to-ground relationship that is required in order to relate a 2d location (pixel) in the image to a corresponding location on the terrain surface, its corresponding predicted accuracy is either not represented at all or not represented appropriately – either in the metadata per se or in vendor documentation. As summarized earlier in this document, reliable predicted accuracy is key to successful applications.

Commodities Example: 3d Point Clouds

Another type of commodities data that is becoming more and more prevalent are 3d Point Clouds which are directly applicable to various geolocation applications. A representative example of a 3d Point Cloud is presented in Figure 12.1-3 that was generated using EO imagery. Many 3d Point Clouds are based on Lidar data instead, and are usually of higher fidelity and higher accuracy. However, for both cases, predicted accuracy is usually either unavailable or not represented appropriately in the corresponding metadata or in vendor documentation.

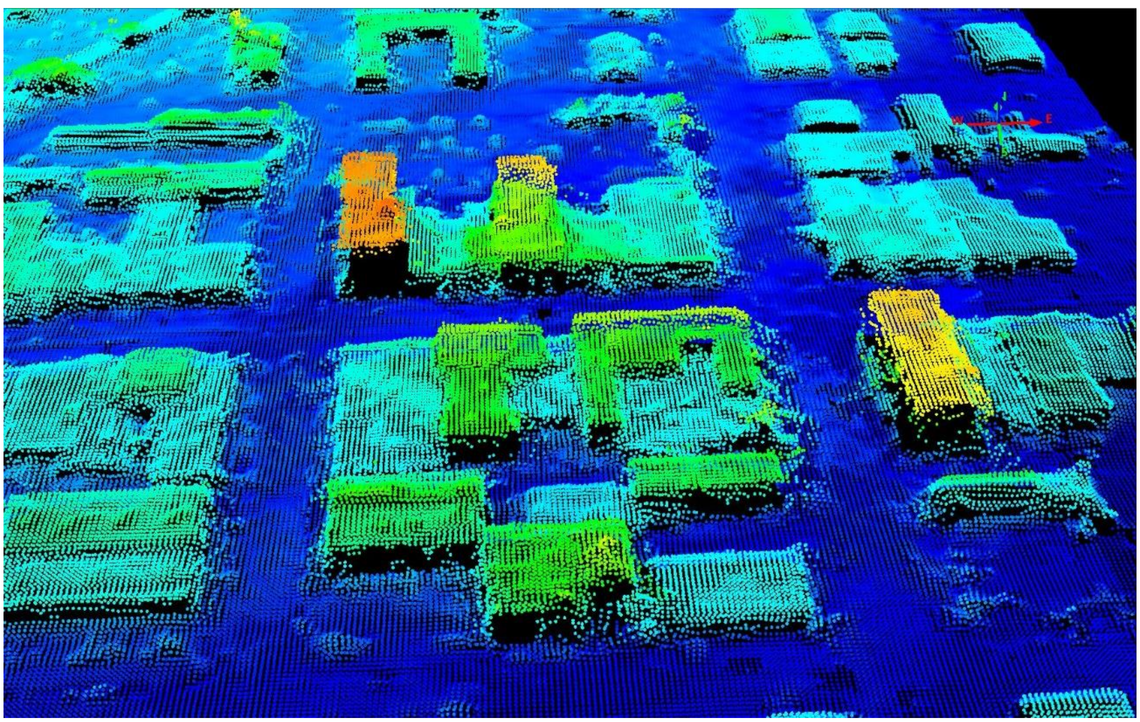


Figure 12.1-3: A portion of a 3d Point Cloud of an urban scene at a specific viewing orientation and zoom-factor; color-coded based on height; generated using aerial imagery from the National Agriculture Imagery Program (NAIP); public domain.

Crowd-Sourcing Example: Digital Maps

A major type of crowd-sourcing data of interest corresponds to digital maps. A representative example is presented in Figure 12.1-4. The digital map appears of reasonable quality, although we do not know: (1)

its geolocation accuracy, (2) whether or not it contains all relevant features and annotations, and (3) whether or not it contains any blunders or mis-information. This is a general problem regarding the use of crowd-sourcing data. One approach for its mitigation is the comparison of different maps from different providers over the same AOI as discussed in TGD 2f.

However, it is also worth pointing out that crowd-sourcing digital maps can be of real value in the NSG, particularly when there are no reliable standard maps of known quality over the AOI that are both available and reasonably current. For example, crowd-sourcing digital maps can be invaluable during humanitarian crises with frequent updates by volunteers over the AOI.

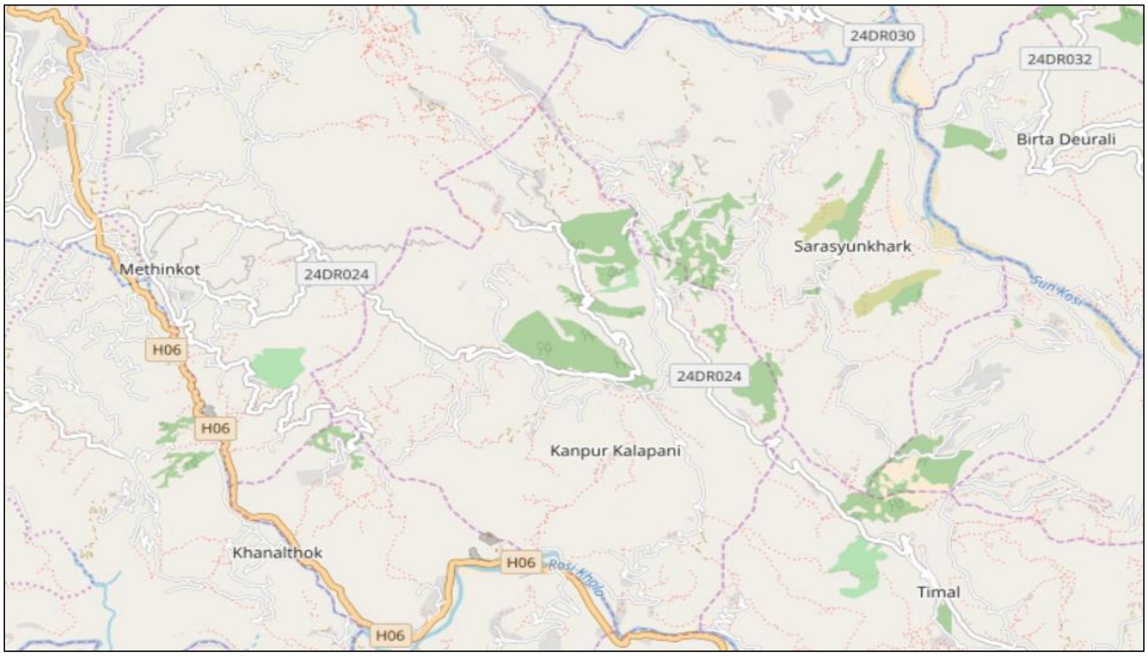


Figure 5.12.1-4: A crowd-sourced digital map via OpenStreetMap (05 July 2018) with a general-use license; an approximately 60 square mile AOI and 15 miles from downtown Kathmandu, Nepal

5.12.2 Information compiled, collated, and modeled

As detailed in TGD 2f, it is recommended that different types of information be compiled, collated, and modeled depending on whether External Data is commodities data or outsourcing data. Figure 5.12.2-1 presents a summary of the correspondence between the types of External Data to the corresponding types of information used to represent their quality. The types of information available to represent crowd-sourcing data is typically limited relative to that available to represent commodities data. As such, the former is less detailed and more empirical.

In general, the overall amount of collective information "grows" as Quality Assessment is continuously performed in the NSG. Also, regardless whether commodities data or crowd-sourcing sourcing data, it is to be further categorized by applicable type or class (category), including vendor, date-range, and general generation technique. It is also to be distributed to appropriate NSG members by the particular organization doing the categorization and related processing.

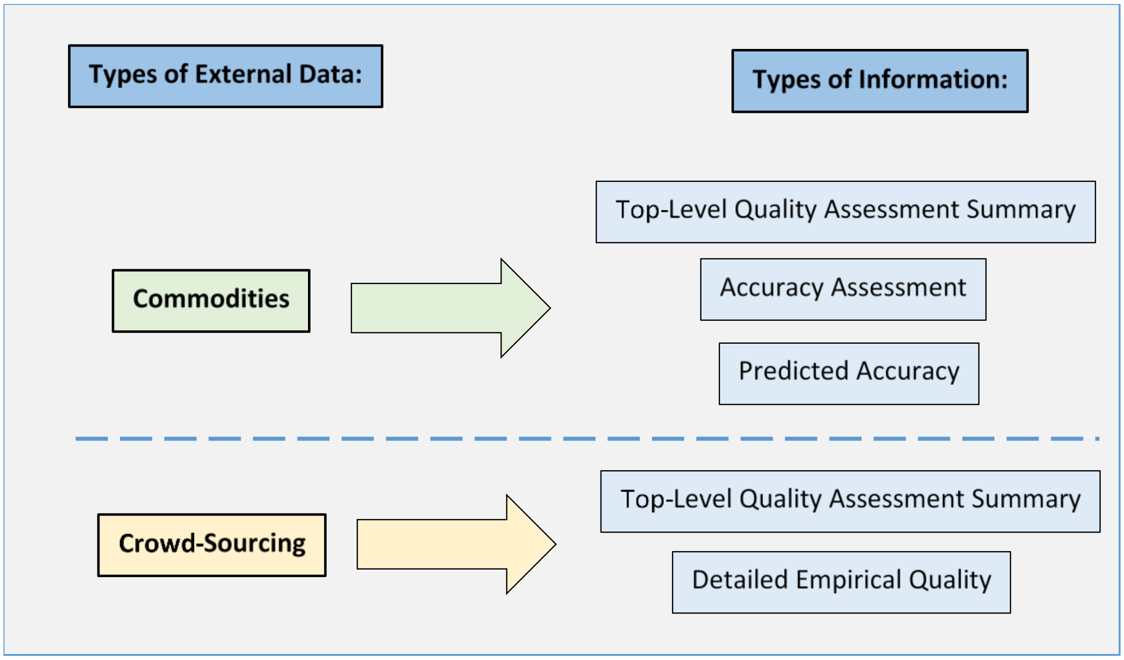


Figure 5.12.2-1: External Data-to-Information correspondence

The accuracy assessments associated with commodities data are contained in populated accuracy assessment models and based on ensemble statistics of geolocation error, usually based on the availability of "ground truth" or related data, such as control imagery, etc. This data, in turn, is used to help populate and refine predicted accuracy models for arbitrary geolocation data or products associated with the type of data, as defined in the previous paragraph.

5.12.3 Predicted Accuracy Models for Commodities Data

Commodities data is further categorized as associated with either geolocation data (e.g., images) or geolocation products (e.g., 3d Point Clouds). The former is data that enables the generation of the latter, which includes geolocations per se. Commodities data is further sub-categorized by the specific type or class of data: vendor, applicable date-range, etc.

Both geolocation data and geolocation products consist of a collection of elements; for example, 2d image (pixel) locations for data and 3d geolocations for products.

5.12.3.1 An important part of a Complete Sensor Model

Regardless if geolocation data or geolocation products, a predicted accuracy model is an important part of an associated complete sensor model. A complete sensor model is needed for useful applications of commodities data in the NSG, as detailed in TGD 2f and summarized as follows.

A complete sensor model consists of:

1) A basic sensor model
2) A predicted accuracy model

3) An optional adjustment model
    a) A correction grid

The basic sensor model provides the ground-to-data function for geolocation data, and the ground-to-product function for geolocation products. Without these functions, the correspondence between a data or product element to a geolocation cannot be made. An example of a data element is an image (pixel) location when data is an image. An example of a product element is a geolocation per se when the product is a 3d Point Cloud.

The predicted accuracy model provides for explicit predicted accuracy of data elements or product elements, as appropriate, and includes corresponding predictive statistics. It also enables (optional) adjustment of the elements via a correction grid if independent information is available.

The basic sensor model is assumed provided by the vendor and its defining parameters provided in the corresponding data's or product's metadata. The predicted accuracy model is almost never provided by the vendor, and is assumed generated and provided by one or more members of the NSG via the techniques detailed in TGD 2f.

<u>Representative examples of the basic sensor model</u>

The basic sensor model for geolocation products is trivial, i.e., the ground-to-product function is essentially the identity function. The basic sensor model for geolocation data consists of the ground-to-data function. A typical example is a rational polynomial for data that corresponds to an image, as illustrated in Figure 5.12.3-1. The polynomial coefficients that define the ground-to-data function are provided by the image vendor as part of the image metadata. The data-to-ground function is its inverse. In this example, a data element corresponds to an image (pixel) location, and the ground-to-data function is simply termed the "ground-to-image function", and its inverse, the "image-to-ground function".

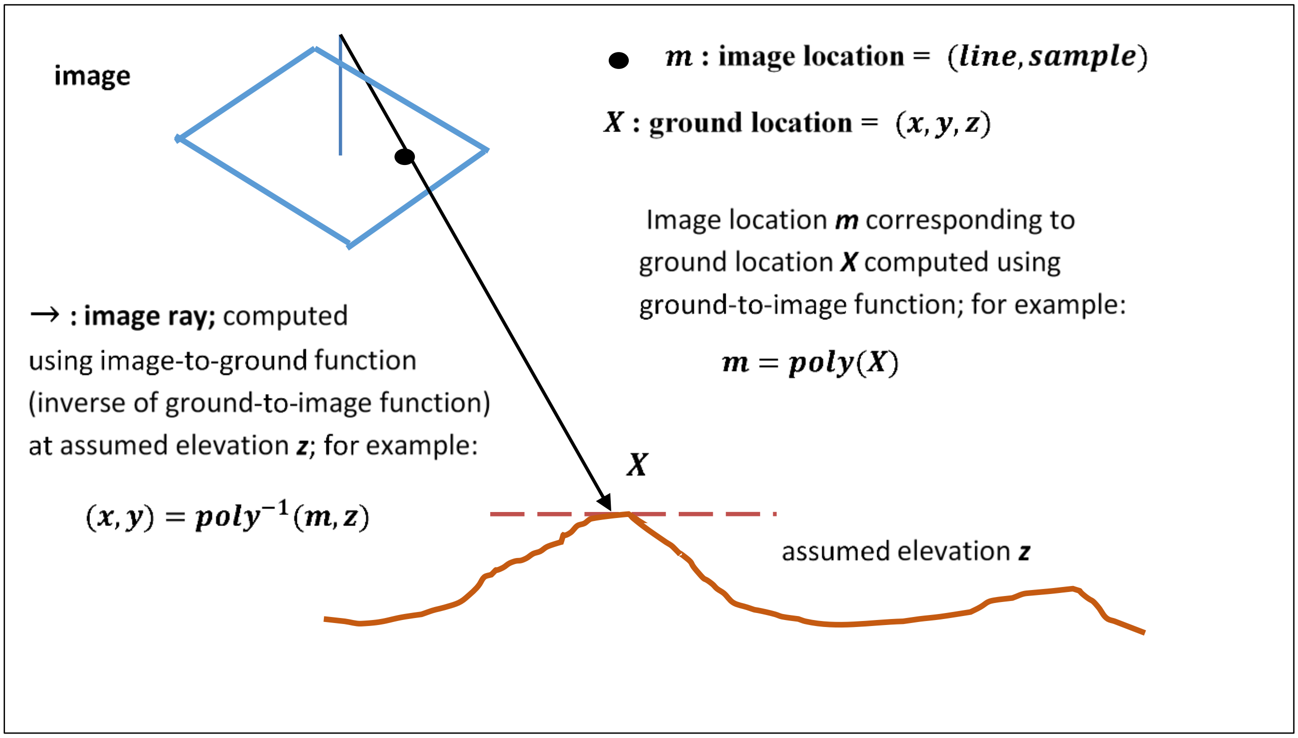


Figure 5.12.3-1: The ground-to-image function provided by the basic sensor model; an estimate of elevation is also required in order to implement its inverse, the image-to-ground function.

Representative example of the predicted accuracy model

A representative example of the predicted accuracy model is presented in Section 5.12.4.

5.12.4 A representative example of a Predicted Accuracy Model for Commodities Data

The predicted accuracy model for a 3d Point Cloud is representative of the predicted accuracy model for commodities data, and for geolocation products in particular. It is relatively simple and consists of the predictive statistics for representation of the errors in the geolocations contained in the 3d Point Cloud as a homogeneous random field. The predictive statistics simply consist of:

- a $3x3$ error covariance matrix
- a few scalar parameters that define the correlation function, an spdcf
    - the spdcf represents the spatial correlation of errors between 3d geolocations in the same realization of the product.

The above predictive statistics (values) are applicable to all geolocations in an arbitrary but specific realization of the product, i.e., the random field is assumed homogeneous.

Population of the model

The predicted accuracy model is populated with values for the above predictive statistics that were typically based on or "tuned" using sample statistics. The sample statistics are contained in a corresponding populated accuracy assessment model, as detailed in TGD 2f. The techniques for the computation of the sample statistics are also “tailored” to the availability of few samples of error, as is typical. However, when ensemble statistics based on a new and independent group of samples are available, the populated accuracy assessment model is updated, and correspondingly, so is the populated predicted accuracy model.

Both the populated accuracy assessment model and the populated predicted accuracy model correspond to the same type or class of 3d Point Cloud, i.e., vendor, date-range, etc. A 3d Point Cloud that “uses” the predicted accuracy model typically did not contribute to the samples of error associated with the corresponding accuracy assessment model.

The populated predicted accuracy model is made available by a member(s) of the NSG, not the vendor, as a supplement to 3d Point Cloud metadata. It is applicable to any 3d Point Cloud identified as corresponding to the associated type or class of 3d Point Cloud, i.e., vendor, date-range, etc.

Suitability of the model for the representation of errors

The following illustrates the suitability of the above predicted accuracy model for 3d Point Clouds by presenting corresponding errors simulated consistent with its predictive statistics. The same predictive statistics are applicable to different realizations of the 3d Point Cloud.

More specifically, Figures 5.12.4-1 and 5.12.4-2 present simulated examples of 3d geolocation errors associated with two different realizations of a 3d Point Cloud that correspond to the same type or class of product (vendor, date range, etc.). Only horizontal errors are presented for clarity across a grid of horizontal locations in a portion of the footprint corresponding to each product realization. The figures are self-scaled with a common scale factor (sf) applicable to all 2d random vectors of horizontal error (blue arrows) in a given figure as quantified in its title.

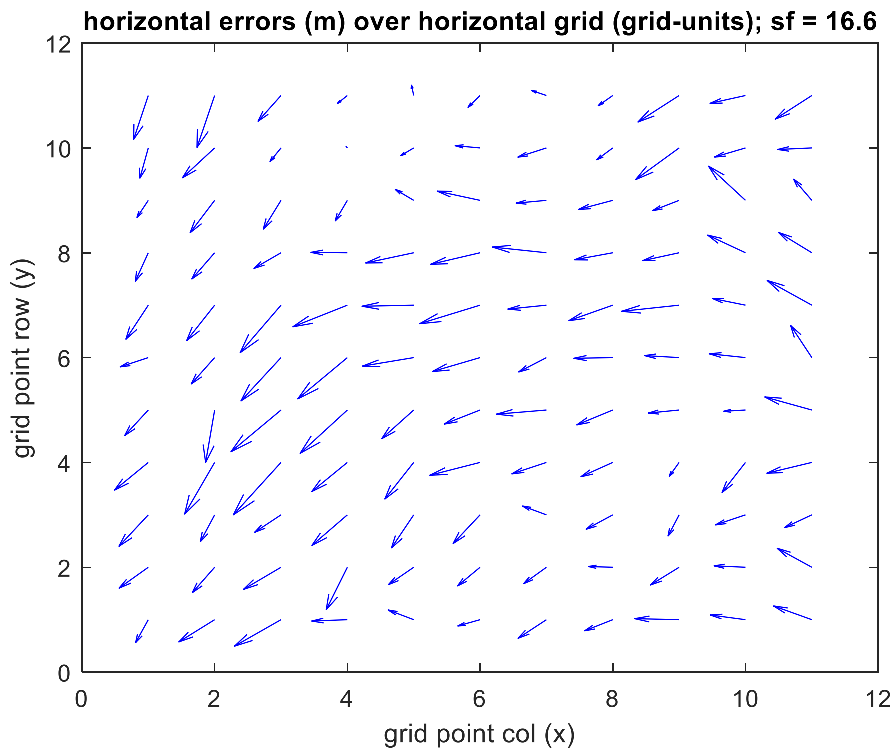


Figure 5.12.4-1: Product realization #1: horizontal errors across a horizontal grid

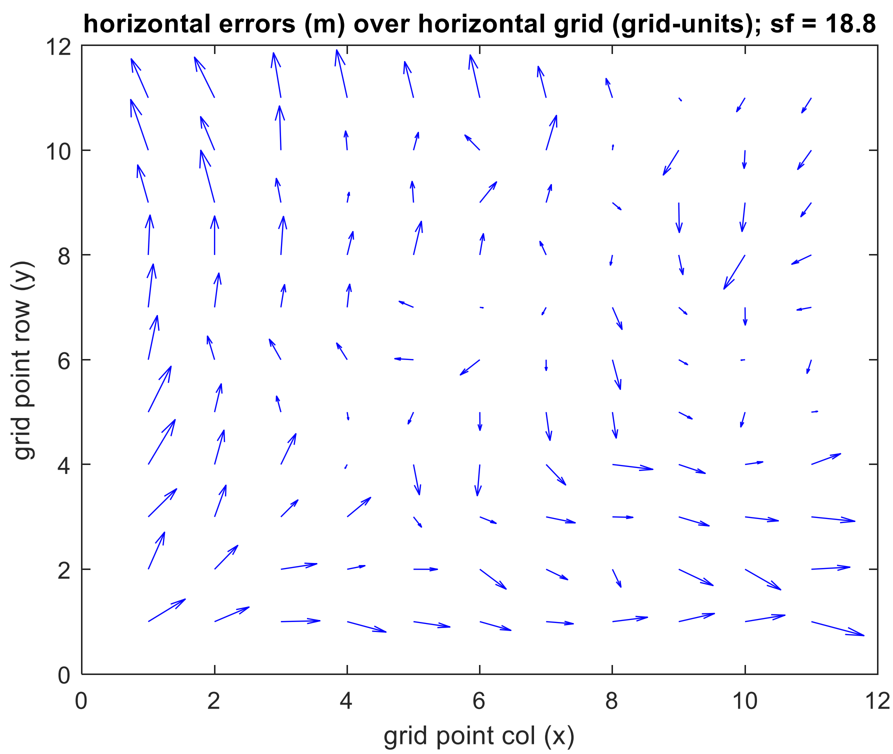


Figure 5.12.4-2: Product realization #2: horizontal errors across a horizontal grid

Although errors vary, their magnitudes are “bounded” by the supplied error covariance matrix and their statistical similarity in the same realization is “bounded” by the supplied spdcf. Note the statistical similarity of errors in the same realization – the closer the locations, the greater their similarity, and correspondingly, the better their relative accuracy.

Furthermore, although the errors across the two realizations look different, they share common statistical characteristics. The reason that they are different is due to the individual 3d Point Clouds being generated using different data of the same type (e.g., different EO images), on different dates, and typically over different regions of the earth.

In general, regardless the specific realization, the predictive statistics can be used to generate the full predicted error covariance for an arbitrary subset of the geolocations in the same product realization, including the predicted relative accuracy between each pair of geolocations. Its availability is critical to many geolocation applications, including adjustment of 3d Point Clouds using other overlapping 3d Point Clouds or control points as detailed in TGD 2f.

Although the above errors were simulated using the techniques of TGD 2e and the supplied predictive statistics, their general characteristics have been verified using numerous examples of real 3d Point Clouds.

Summary

In summary, this section of the document summarized the baseline predicted accuracy model for geolocation products, 3d Point Clouds in particular. TGD 2f also details an optional, new, and higher fidelity model that is based on a Mixed Gaussian Random Field. It allows for the variation of predictive statistics over a product and is still convenient for the user or application of a 3d Point Cloud to implement.

Note: errors (2d random vectors) between grid point locations in Figures 5.12.4-1 and 5.12.4-2 can be computed by bilinear interpolation of errors at surrounding grid point locations. This is consistent with the recommended method for the computation of geolocations between grid point locations by bilinear interpolation of geolocations at surrounding grid point locations. The use of bilinear interpolation also assumes that the four locations are in approximately the same plane in a local tangent plane coordinate system.

5.13 Provenance for Predicted Accuracy

As a subset of the provenance of NSG internal data and its products, geolocations and their predicted accuracies require, as a minimum, corresponding “time tags” to specify the time associated with their generation and the time-of-applicability of the data used to generate them. Thus, for example, if a set of imagery was used to generate feature geolocations and their predicted accuracies, the appropriate imaging time(s) should also be specified. Thus, if the features are then utilized two years later, it is known that their accompanying geolocations and their predicted accuracies are applicable two years (or more) earlier. Correspondingly, predicted accuracies can be “degraded” (e.g. accompanying error covariances inflated) or flagged as “do not use”, if necessary, in order to account for any subsequent earthquakes, landslides, urban development, etc., known to have occurred.

Coordinate systems and their datum

In addition to the association of times to geolocation products and data, relevant geodetic coordinate systems and their datum should also be identified. Newer versions of a datum may compensate for more recent polar wander, plate tectonics, and other geodetic phenomena.

Identification of applicable geodetic coordinate systems and their datum also allows for the degradation (inflation) of predicted accuracy for potentially one or more of multiple sets of products and/or data that use different representations for geolocations in a mixed application. The inflation of predicted accuracy corresponds to expected differences in their representations.

Identification of geodetic coordinate systems and their datum can also allow for the conversion of geolocations using different representations to a common and more recent representation. Conversions applied to an earlier representation should consist of changes (translations) due to estimated geodetic changes that occurred from the earlier representation to the more recent representation. Future research is recommended for both the conversion of representations and the inflation of predicted accuracy due to either different representations or to imperfect conversions.

Regarding differences between different representations of geolocations per se, changes in the WGS 84 reference datum, i.e., a different realization of WGS 84, can be significant for some applications. In 1994, DoD introduced a realization of WGS 84 that was based completely on GPS observations, instead of Doppler observations. This realization is officially known as WGS 84 (G730) where the letter G stands for "GPS" and "730" denotes the GPS week number (starting at 0h UTC, 2 January 1994) when the National Imagery and Mapping Agency (NIMA) started expressing their derived GPS orbits in this frame. The realization of WGS 84, adopted in 20 January 2002, is termed WGS 84 (G1150). (See reference [14] for more details.) It is recommended that the most recent WGS 84 (GXXXX) realization be used for applications when possible, i.e. used for the current generation of products and data, and possibly used as the “master” representation for conversions to a common representation in a mixed application that was discussed in the paragraphs above.

In addition to more specifics regarding the above degradation of predicted accuracy and conversions between geolocation representations for a specific application, we recommend that a general area of research should include whether these should be done, and if so how, for NSG “data bases” in general. More specifically, determine whether earlier representations of geolocation products and data need to be brought “up to-date” or not, and if so, specifically how. Appropriate deterministic and statistical-based decision processes as well as equations for the degradation of predicted accuracy and/or geolocation conversions need to be developed and implemented consistently throughout the NSG.

5.14 Computer System Capabilities

Due to tremendous advances in computer systems over the last few decades, approaches related to error modeling have expanded significantly: complex systems can be effectively simulated via Monte-Carlo methods; very large multi-state vector error covariance matrices can be generated, stored, and disseminated; estimation algorithms can correspond to non-linear estimation; and analytic approximations can be replaced by straight-forward numerical integration. This document and the TGD 2 documents, in part, reflect these expanded capabilities in the methods and algorithms that they present.

5.15 Recommended Practices Overview

The companion TGD 2 documents include recommended standard practices or methodologies regarding Accuracy and corresponding Error Modeling, applicable throughout all relevant modules in the NSG (see Figure 4.1-1). In this introductory guidance document, TGD 1, we also gave an overview of many of these practices, which are summarized and categorized at three different levels for an NSG Geolocation System in Tables 5.15-1 through 5.15-3:

Table 5.15-1: Recommended practices for an NSG Geolocation System at the system level

Recommended Practices
Level: High (system level)
Statistical Error Models are implemented: Statistical error models are defined and utilized within each main module (Collection, Value-Added Processing, Exploitation) and transferred among main modules in a Geolocation System as appropriate. An appropriate statistical error model is a necessary condition for optimal system accuracy and reliable predicted accuracy under various conditions.
Specification and Validation of Predicted Accuracy is included: Specification and validation of a Geolocation System's Accuracy requirements are accompanied by the specification and validation of its Predicterd Accuracy requirements as well.
Validation of Requirements is based on an adequate number of Independent Samples: Validation is based on sample statistics with enough independent samples for a specified level of statistical significance. This is particularly important for errors that are appropriately represented as, or include the effects of, stochastic processes or random fields. Samples must be taken (pooled) over multiple time or spatial intervals that are widely separated relative to temporal or spatial correlations.
Externally generated data requires the assessment of its Accuracy: The above entries are directed at NSG-internal modules and data. Externally generated data, such as crowd-sourcing and commodities data, require different, lower fidelity, but never the less as important processing. This processing is essentially limited to the asessment of it accuracy and its quality assessment, not the formal validation of accuracy
Provenance for predicted accuracies is included: Provenance for predicted accuracies are to be generated, maintained, and utilized.
Standard Application Program Interfaces are recommended: Standard Application Program Interfaces are recommended for all modules.

Table 5.15-2: Recommended practices for an NSG Geolocation System at the module level

Recommended Practices
Level: Medium (system module level)
Full Error Covariance Matrix utilized: The full error covariance matrix is utlized within and made availabe between modules.
Error Covariance Matrix is not replaced by summary statistics: The (full) error covariance matrix is not replaced by summary metrics, such as predictive scalar accuracy metrics CE and LE. These scalar accuracy metrics do serve a useful purpose, but supplement, not replace, the error covariance matrix.
Estimators make appropriate use of statistical error models Estimators (Weighted Least Squares, Kalman filters, etc.) make appropriate use of statistiscal errors models in order to perform rigorous error propagation and weight its various measurements appropriately. They also generate a reliable predicted accuracy for an arbitrary but specific solution.
Estimators perform QC on their solutions: Estimators perform Quality Control (QC)on their solution based on Quality Assurance (QA) requirements for the estimator.
Periodic Calibration is performed: Periodic calibration of accuracy, predicted accuracy, and statistical error models is performed, typically using ground truth or surveyed geolocations.
Monte Carlo Simulation is utilized as appropriate: Rigorous error propagation and the analysis of the effects of errors in complex geolocation systems can be effectively performed using Monte Carlo simulation of errors. Corresponding Monte Carlo simulation of errors can also be embedded in the generation of various geolocation products when appropriate.

Table 5.15-3: Recommended practices for an NSG Geolocation System at the intra-module level

Recommended Practices
Level: Low (intra module level)
Error Ellipsoids are uitilized: Error ellipsoids as well as confidence ellipsoids at speciified levels of probability and based on the error covariance matrix are utilized and made available to the human operator or analyst whenever feasible and appropriate.
Scalar accuracy and predicted accuracy summaries based on LE, CE, and SE are utilized: Scalar accuracy and predicted accuracy summaries based on LE, CE, and SE at specified levels of probability are utilized as appropriate as convenient summaries of accuracy and predicted accuracy. They are computed using the algorithms presented in TGD 2a as a predictive statistic and in TGD 2b as a sample statistic.

Document Summary

This document presented an integrated overview of recommended methodologies, procedures, and algorithms, such that geospatial accuracy is close to optimal for arbitrary geolocations generated or extracted by an NSG Geolocation System, specific geolocations are accompanied by reliable predicted accuracy, and such that capabilities can be specified, validated, verified, and assessed. TGD 2a – 2f provide corresponding and additional details.

6 Notes

6.1 Intended Use

This information and guidance document provides technical guidance to inform the development of geospatial data accuracy characterization for NSG GEOINT collectors, producers and consumers -- accuracy characterization as required to describe the trustworthiness of geolocations for defense and intelligence use and to support practices that acquire, generate, process, exploit, and provide geolocation data and information based on geolocation data. This document is part of a series of complementary documents. TGD 1 provides an overview to more detailed topical technical guidance provided in TGD 2a – TGD 2f on the subjects of predictive statistics, sample statistics, specification and validation, estimators and quality control, Monte-Carlo simulation, and External Data and quality assessment.

7 References

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[2] Bresnahan, P., et al, "Planet Dove Constellation Absolute Geolocation Accuracy, Geolocation Consistency, and Band Co-Registration Analysis", Joint Agency Commercial Imagery Evaluation (JACIE) Workshop, 19 – 21 September 2017, PA case # 17-582; https://calval.cr.usgs.gov/wordpress/wp-content/uploads/Paul-Bresnahan-1.pdf.

[3] Dolloff, John, "Introduction to Photogrammetric-based Geopositioning", April 2010.

[4] Dolloff, J. T., "The full multi-state vector error covariance matrix: Why needed and its practical representation", Proceedings of the SPIE Defense and Security Conference, 2013.

[5] Dolloff, J., Braun, A., and Theiss, H., "Generalization of the SPDCF Method for the Assembly of Multi-State Vector Covariance Matrices", NGA white paper, PA case #18-156, 29 November 2017.

[6] Dolloff, John, and Carr, Jacqueline, "Methods for the Specification and Validation of Geolocation Accuracy and Predicted Accuracy", Proceedings of the SPIE Defense and Commercial Sensing Conference, 2017.

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[11] Doucette, P., Dolloff, J., Lenihan, M., "Geostatistical modeling of uncertainty, simulation, and proposed applications in GIScience", Proceedings of the SPIE Defense and Security Conference, 2015.

[12] Gelb, A., Applied Optimal Estimation, The M.I.T Press, 1974.

[13] Mikhail E., Bethel, J., and McGlone C., Modern Photogrammetry, John Wiley and Sons, 2001.

[14] NOAA, "Vertical Datum Transformation, a tutorial"; https://vdatum.noaa.gov/docs/datums.html; accessed 20 January 2020.

[15] Papoulis, A., Probability, Random Variables, and Stochastic Processes, 3rd edition, McGraw-Hill, 1991.

[16] Planet Lab's website; www.planet.com/gallery/el-alamein-20160828/; accessed 02 July 2018.

Appendix A: Additional Terms and Definitions

There are a number of authoritative guides as well as existing standards within the NSG and Department of Defense for definitions of the identified additional terms used in this technical guidance document. In many cases, the existing definitions provided by these sources are either too general or, in some cases, too narrow or dated by intended purposes contemporary to the document's development and publication. The definitions provided in this document have been expanded and refined to explicitly address details relevant to the current and desired future use of accuracy in the NSG. To acknowledge the basis and/or linage of certain terms Section 3.1, we reference the following sources considered as either foundational or contributory:

[a] Anderson, James M. and Mikhail, E., Surveying: Theory and Practice, 7th Edition, WCB/McGraw-Hill, 1998.

[b] DMA-TR-8400.1, DMA Technical Report: Error Theory as Applied to Mapping, Charting, and Geodesy.

[c] Defense Mapping Agency, Glossary of Mapping, Charting, and Geodetic Terms, 4th Edition, Defense Mapping Agency Hydrographic/Topographic Center, 1981.

[d] ISO TC/211 211n2047, Text for ISO 19111 Geographic Information - Spatial referencing by coordinates, as sent to the ISO Central Secretariat for issuing as FDIS, July 17, 2006.

[e] Joint Publication (JP) 1-02, Department of Defense Dictionary of Military and Associated Terms, November 8, 2010 as amended through January 15, 2016.

[f] MIL-HDBK-850, Military Handbook: Glossary of Mapping, Charting, and Geodetic Terms, January 21, 1994.

[g] MIL-STD-2401, Department of Defense Standard Practice; Department of Defense World Geodetic System (WGS), January 11, 1994

[h] MIL-STD-600001, Department of Defense Standard Practice; Mapping, Charting and Geodesy Accuracy, February 26, 1990.

[i] *National System for Geospatial Intelligence* [Brochure] Public Release Case #15-489.

[j] NGA.STND.0046_1.0, The Generic Point-cloud Model (GPM): Implementation and Exploitation, Version 1.0, October 03, 2015.

[k] Oxford Dictionaries (www.oxforddictionaries.com/us/) copyright © 2016 by Oxford University Press.

[l] Soler, Tomas and Hothem, L., “Coordinate Systems Used in Geodesy: Basic Definitions and Concepts”, Journal of Surveying Engineering, Vol. 114, No. 2, May 1988.

A priori - Relating to or denoting reasoning or knowledge that proceeds from theoretical deduction rather than from observation or experience. [k]

- For typical NSG accuracy and predicted accuracy applications, *a priori* refers to a mathematical statistical model of errors and/or the corresponding state vector containing those errors prior to its adjustment using additional information.

A posteriori - Relating to or denoting reasoning or knowledge that proceeds from observations or experiences to the deduction of probable causes. [k]

- For typical NSG accuracy and predicted accuracy applications, *a posteriori* refers to a refined mathematical statistical model of errors and/or the corresponding state vector containing those errors following its adjustment using additional information.

Absolute Horizontal Accuracy - The range of values for the error in an object's horizontal metric geolocation value with respect to a specified geodetic horizontal reference datum, expressed as a radial error at the 90 percent probability level (CE). [b],[f],[j]

- There are two types of absolute horizontal accuracy: *predicted* absolute horizontal accuracy is based on error propagation via a statistical error model; and *measured* absolute horizontal accuracy is an empirically derived metric based on sample statistics.
- The term "horizontal accuracy" is assumed to correspond to "absolute horizontal accuracy".
- The 90% probability level (CE) is the default; 95% and 50% probability levels are optional, i.e., CE_95 and CE_50, respectively.

Absolute Vertical Accuracy - The range of values for the error in an object's metric elevation value with respect to a vertical reference datum, expressed as a linear error at the 90 percent probability level (LE). [b],[f],[j]

- There are two types of absolute vertical accuracy: *predicted* absolute vertical accuracy is based on error propagation via a statistical error model; and *measured* absolute vertical accuracy is an empirically derived metric based on sample statistics.
- The term "vertical accuracy" is assumed to correspond to "absolute vertical accuracy".
- The 90% probability level (LE) is the default; 95% and 50% probability levels are optional, i.e., LE_95 and LE_50, respectively.

Accuracy (augmented definition) - The range of values for the error in an object's metric value with respect to an accepted reference value expressed as a probability. [f]

In an NSG Geolocation System a typical object of interest is an arbitrary 3d geolocation extracted by the system, with a more specific definition of accuracy as follows:

- Accuracy
  - The probability of error corresponding to an arbitrary 3d geolocation extracted by the system. The probability of error is typically expressed as CE90=XX meters, the 90% probability that horizontal circular or radial error is less than XX meters, as well as LE90=YY meters, the 90% probability that vertical linear error is less than YY meters. In general, the error is represented as a 3d random vector and its corresponding CE90 and LE90 values are typically specified and/or evaluated based on sample statistics of independent samples of error.
    - The accuracy requirements for a Geolocation System are typically specified as horizontal radial error and vertical linear error of an arbitrary but specific 3d geolocation are less than specCE90 with a probability of 90% and less than specLE90 with a probability of 90%, respectively.

An "accurate geolocation" is defined as the geolocation of a specific extraction that satisfies the specified accuracy requirements of the Geolocation System.

Accuracy Assessment Model - A collection of sample statistics that, when populated, characterizes the geolocation accuracy or sensor measurement accuracy of a specific data/product or a collection of data/products that correspond to the same type, such as an image/metadata or a geolocation product from a specific provider, date-range, etc. In this series of technical guidance documents, an accuracy assessment model typically corresponds to External Data, and commodities data, in particular.

There are two categories of accuracy assessment models: (1) Geolocation Product and (2) Geolocation Data. The former corresponds to geolocation products per se, such as 3d Point Clouds, and the latter corresponds to geolocation data that can be used to generate geolocation products, such as an image and its metadata. A populated accuracy assessment model is typically used for the population of a predicted accuracy model.

Bias Error - A category of error; an error that does not vary from one realization (trial or experimental outcome) to the other. When error is represented as a random variable, random vector, stochastic process, or random field, a bias error corresponds to a non-zero mean-value. [f],[j]

- Caution: a given realization of a mean-zero stochastic process with typical temporal correlation and over a reasonable finite time interval appears to have a non-zero sample mean-value; however, when sample statistics are taken over enough multiple (independent) realizations, the sample mean-value approaches zero in accordance with the true mean-value. This characteristic extends to random fields as well.

CE-LE Error Cylinder - A 3D cylinder made up of CE and LE such that there is between 81-90% probability that the 3d error resides within.

Confidence Ellipsoid - An ellipsoid centered at an estimate of geolocation such that there is a 90% probability (or XX% if specified specifically) that the true geolocation is within the ellipsoidal boundary (ellipsoid interior). A confidence ellipsoid is typically generated based on an error covariance matrix, an assumed mean-value of error equal to zero, and an assumed multi-variate Gaussian probability distribution of error in up to three spatial dimensions.

Correlated Error - A category of errors; errors that are correlated with other errors, and typically represented in the NSG as a random vector, stochastic process, or random field. A correlated error is independent (uncorrelated) with itself and other errors from one realization (trial or experimental outcome) to the next. However, within a given realization, it is correlated with other errors of interest:

- If a random vector, the various elements (random variables) which make it up are correlated with each other (intra-state vector correlation).
- If a stochastic process, the collection of random vectors which make up the stochastic process are correlated with each other (inter-state vector correlation). That is, the elements of one random vector are correlated with the elements of another random vector, typically the closer the two random vectors in time, the greater the correlation. A similar concept is applicable to random fields.

Correlated Values - Values (of random variables) which are related by a statistical interdependence. For two random variables, this interdependence is represented by their covariance and typically expressed as a correlation coefficient – both have non-zero values. This interdependence is relative to deviations about their respective mean-values. [f]

Covariance - A measure of the mutual variation of two random variables, where variations (deviations or dispersions) are about their respective mean-values. If the covariance between two random values is zero, they are uncorrelated. [b]

Covariance Function - The cross-covariance matrix of two random vectors associated with a (same) stochastic process or random field as a function of their corresponding time or spatial locations, respectively. If the stochastic process is (wide sense) stationary or the random field (wide sense) homogeneous, the cross-covariance matrix is a function of delta time or delta position, respectively. When evaluated at delta equal to zero, it equals the common covariance matrix.

Covariance Matrix - A symmetric, nxn positive definite matrix populated with the variances and covariances of the random variables contained within a single, multi-component ($nx1$) state vector or random vector. Note that if row i ($1 \leq i \leq n$) and all corresponding columns j ($1 \leq j \leq n$,$j \neq i$) are zero, random variable i is uncorrelated with all of the other random variables j. [b]

Cross-covariance Matrix - An nxm matrix containing the covariance between each pair of elements (random variables) of an $nx1$ random vector and an $mx1$ random vector.

Deterministic Error - An error that is not random or dependent on "chance" – a "known" value, such as the specific realization of an error of an estimated geolocation as compared to "ground truth", i.e., their difference, where "ground truth" is assumed error-free.

Directed Percentile - The percentile of error along a specified direction, i.e., a directed XX percentile is an $nx1$ vector along a specified direction in n-dimensional space with a magnitude equal to the XX percentile of error along the specified direction.

- For example, a directed 90th percentile of error is an $nx1$ vector $Xdp = r_{1,90}\eta$, where its magnitude $r_{1,90}$ is the 90th percentile of error and η is an $nx1$ unit vector in the specified direction of interest. More specifically, $prob\{|\eta^T \epsilon X| \leq r_{1,90}\} = 0.90$, where ϵX is an arbitrary $nx1$ random error vector associated with the error process of interest. $\eta^T \epsilon X$ is a scalar equal to the component of error in the direction of interest.
- The units of X, its error ϵX, and Xdp are common and typically meters for each component or coordinate; hence, the units of $r_{1,90}$ are meters.
- A directed percentile of error is usually computed as a predictive statistics and based on the error covariance matrix of $nx1$ errors assumed to be (multi-variate) Gaussian distributed.

Earth Centered Earth Fixed Cartesian Coordinate System - The Conventional Terrestrial Reference System (CTRS) with the following definition:

1) Origin: at the geocenter (center of mass of the earth).
2) z-axis: Directed toward the conventional definition of the North Pole, or more precise, towards the conventional terrestrial pole as defined by the International Earth Rotation Service (IERS).
3) x-axis: Passes through the point of zero longitude (approximately on the Greenwich meridian) as defined by the IERS.
4) y-axis: forms a right-handed coordinate system with the x- and z-axes. [l]

Elevation - Vertical distance above a datum, usually mean sea level, to a point or object on the Earth's surface; not to be confused with altitude which refers to points or objects above the Earth's surface. In geodetic formulas, elevations are heights: *h* is the height above the ellipsoid; *H* is the height above the geoid or local datum. Occasionally *h* and *H* may be reversed. See definition of **Height** in TGD 1G (Glossary) for further information. [c],[f]

Error (augmented definition) - The difference between the observed or estimated value and its ideal or true value. [f] There are a number of different categories of errors applicable to the NSG: Bias Error, Random Error, and Correlated Error. In general, an error of interest may be a combination of errors from these categories. Their combination is typically represented as either a random variable, random vector, stochastic process, or random field:

- A random variable represents a bias error plus a random error. The former corresponds to the random variable's mean-value, and if equal to zero, the random variable represents random error only, which is uncorrelated from one realization of the random variable to the next realization.
- A random vector, stochastic process, and random field can represent all three categories of error. The random variables that make-up (are elements of) random vectors are uncorrelated from one realization to the next by definition. However, within a given realization, they can also be correlated with each other:
  - For a random vector per se, this correlation is also termed "intra-state vector correlation".
  - For a stochastic process, which consists of a collection of random vectors, random variables in one random vector can also be correlated with random variables in another random vector, this is also termed "inter-state vector" correlation. The same concept is applicable to random fields.

Error Ellipsoid - An ellipsoid such that there is a 90% probability (or XX% if specified specifically) that geolocation error is within the ellipsoidal boundary (ellipsoid interior). An error ellipsoid can be generated based on a predictive or sample-based error covariance matrix, centered at an assumed predictive mean-value of error equal to zero or a sample-based mean-value of error not equal to zero, and an assumed multi-variate Gaussian probability distribution of error in up to three dimensions.

Estimator - An algorithm/process which estimates the value of an nx1 state vector. Its inputs are measurements related to the state vector and may include *a priori* information about the state vector.

- An estimator is usually designed to be an optimal estimator relative to a cost function, such as the sum of weighted *a posteriori* measurement residuals, minimum mean-square solution error, etc.

- Estimators are sequential or batch processes, and an optimal estimator should include both an estimate of the state vector and its predicted accuracy, usually an error covariance matrix, as output. A properly implemented MIG for a target's geolocation is an optimal estimator.

Gaussian (or Normal) probability distribution - A specific type of probability distribution for a random variable. The distribution is specified by either a Gaussian probability density function or a Gaussian cumulative distribution function. These in turn are completely characterized by the random variable's mean-value and variance.

- The Gaussian (probability) distribution is a common distribution that approximates many kinds of errors of interest to the NSG, and approximates the distribution for a sum of errors from different (non-Gaussian) distributions as well (Central Limit Theorem). A Gaussian distribution corresponding to an nx1 random vector is termed a multi-variate Gaussian distribution.

Geodetic Coordinate System - Coordinate system in which position is specified by geodetic latitude, geodetic longitude and (in the three-dimensional case) ellipsoidal height [d].

Ground Truth - The reference or (assumed) true value of a geolocation of a measured quantity (e.g. associated with an absolute geolocation, or a relative mensuration).

Homogeneous - A descriptor for a random field. A random field is (wide-sense) homogeneous if corresponding (*a priori*) statistics are invariant to spatial location. For example, the mean-value and covariance matrix corresponding to its random vectors are constant, and correlation between two corresponding but arbitrary random vectors in the same realization is a function of spatial distance between them, not the explicit spatial location of each.

Horizontal Error - As applied to geospatial measurements and processes, horizontal error is typically observed in the x, y plane of a local right-handed coordinate system where the x, y plane is defined as tangent to the defined reference surface at the point of origin. While horizontal error is the x and y components of error, it may be generalized by its magnitude or 2D radial error.

Inter-state vector correlation - The correlation between the errors (random variables) of the elements in two different state vectors.

Intra-state vector correlation - The correlation between the errors (random variables) of different elements in the same state vector.

Local Tangent Plane Coordinate System (Coordinate System/Coordinate Reference System) - A local X,Y,Z right-handed rectangular coordinate system such that the origin is any point selected on a given reference ellipsoid, its XY plane is tangent to the reference ellipsoid at the point of origin, and the Y-axis is typically directed to the North Pole (an East-North-Up (ENU) system). [a]

Mean-Value - The expected value of a random variable. Given a collected sample of measurements, the sample mean-value is the average of the values of the sample measurements. The mean-value of a predictive error is typically assumed zero unless specifically stated otherwise. If correctly modeled, the predictive mean-value should be closely approximated by the sample mean-value taken over a large number of independent and identically distributed samples.

- The concept of mean-value readily extends to random vectors and is the vector of the mean-values of the individual components or random variables making up the random vector. It readily extends to stochastic processes and random fields as well, since they are collections of random vectors. If (wide-sense) stationary or (wide-sense) homogeneous, respectively, their corresponding mean-value is a constant random vector mean-value.

Metadata - Higher level or ancillary data describing a collection of data, e.g., the sensor support data corresponding to an image, which specifies corresponding sensor position, attitude, interior orientation parameters, etc.

Multi-Image Geopositioning (MIG) - An optimal solution for a "target's" geolocation (state vector) with reliable predicted accuracies based on the (weighted) measurements of the geolocation in one or more images. A batch process which minimizes the sum of weighted *a posteriori* measurement residuals, where the latter may also include measurements equivalent to *a priori* estimates of geolocation. MIG can also correspond to the simultaneous solution for the geolocation of multiple targets. In general, a MIG solution's predicted accuracies correspond to or are derived from the solution's *a posteriori* error covariance matrix.

Multi-State Vector Error Covariance Matrix - An error covariance matrix corresponding to multiple state vector errors (random error vectors) "stacked" one on top of the other as one large state vector error (random error vector), e.g. to represent the position and attitude errors of multiple images' adjustable parameter errors that impact the solution and predicted accuracy of a subsequent MIG. The multi-state vector error covariance matrix is sometimes termed the joint covariance matrix for a collection of multiple state vector errors.

Order Statistics - Nonparametric statistics performed on a set ordered by ascending magnitude of randomly sampled values. Nonparametric statistics assume no *a priori* information about the underlying probability distribution of a random variable such as its mean-value, variance, or type of probability distribution function. In the NSG, order statistics are used to compute scalar accuracy metrics from independent and identically distributed samples of error.

Percentile - If a random variable's probability (or sample) distribution is divided into 100 equal parts, the value of the random variable that corresponds to the percentage of the distribution equal to or below the specified percentile, e.g. the 90th percentile indicates the lowest sample value such that it is greater than the values of 90 percent of the samples.

- A more formal definition is as follows: The p percentile of a random variable x is defined as the smallest number x_p such that $p = prob\{x \leq x_p\}$. Thus, the probability distribution function (typically unknown) of the random variable x evaluated at x_p is equal to p. x_p is a deterministic parameter with typically unknown value.

Precision - The closeness to one another of a set of repeated observations of a random variable. [a],[f]

- In terms of accuracy, precision is a measure of the repeatability of the underlying errors. High accuracy implies high precision, but not vice versa. For example, for an error represented as a random variable, high precision implies a small standard deviation, but high accuracy implies both a small standard deviation and a small or zero mean-value (or bias).

Predicted Accuracy (augmented definition) – The range of values for the error in a specific object's metric value as expressed by a statistical or predictive error model, and may also be expressed as a probability if a specific probability distribution is specified or assumed, typically a Gaussian (or Normal) probability distribution.

In an NSG Geolocation System a typical object of interest is an arbitrary but specific 3d geolocation extracted by the system, with a corresponding definition of predicted accuracy as follows:

- Predicted accuracy
    - A statistical description of the error in a specific geolocation extracted by the system. The error is expressed as a 3d random vector and the statistical description consists primarily of an error covariance matrix of the random vector about a mean-value typically assumed equal to zero unless specifically stated otherwise. The probability of error can also be computed if either a probability distribution is also specified, or a multi-variate Gaussian probability distribution of error is assumed. The probability of error is expressed as a probability or confidence ellipsoid at a specified probability or confidence level, respectively, and may also be expressed as CE90 and LE90.
        - The estimate of geolocation is usually performed by an estimator, such as a Weighted Least Squares estimator, with a corresponding solution error that is a function of measurement errors that are random from one solution or realization to the next as well as sensor-to-ground geometry at different geolocations.
        - The term "predicted" in predicted accuracy does not correspond to a prediction of accuracy applicable to the future since the corresponding error corresponds to a geolocation already generated or extracted by the NSG Geolocation System.
        - "Reliable predicted accuracy" is defined as predicted accuracy that is consistent with solution error(s).
    - An exception to the above is as follows: If so caveated, predicted accuracy can also correspond to a hypothetical extraction of a specific geolocation, such as that in support of sensor tasking. The extraction makes use of specific, but hypothesized, sensor-to-geolocation geometry, and the same extraction algorithm and *a priori* error models as would be used for an actual (operational) extraction. No actual measurements are incorporated, and measurements are either simulated or not used at all. If the latter, only predicted accuracy is computed by the extraction algorithm, not the geolocation.

Predicted Accuracy Model - A collection of predictive statistics that characterize the geolocation accuracy or related sensor measurement accuracy in an arbitrary data/product of a specified type. When a populated predicted accuracy model is assigned to a specific data/product, it becomes its predicted accuracy. In this series of technical guidance documents, a predicted accuracy model typically corresponds to External Data, and commodities data, in particular.

There are two categories of predicted accuracy models: (1) Geolocation Product and (2) Geolocation Data, the latter subcategorized by Sensor-space and Measurement-space. A predicted accuracy model is typically populated based on a corresponding populated accuracy assessment model.

Principal Matrix Square Root - The principal matrix square root of a valid error covariance matrix is a valid error covariance matrix itself of the same dimension such that when multiplied with itself yields the original error covariance matrix. The calculation of principal matrix square root is based on Singular Value Decomposition.

Probability density function (pdf) - A function that defines the probability distribution of a random variable. If continuous, its integral is the (cumulative) probability distribution function.

Probability distribution - Identifies the probability of a random variable's values over an applicable range of values. There are many different types of probability distributions: Gaussian or Normal, uniform, exponential, etc.

- In most NSG applications for accuracy and predicted accuracy, the random variable and its probability distributions are assumed continuous.
- The probability distribution is specified by either a probability density function or a (cumulative) probability distribution function; either based on an *a priori* model or sample statistics.

Probability distribution function - The (cumulative) probability distribution function defines the probability that a random variable's value is less than or equal to a specified number in the interval [0,1].

Provenance - The place of origin or generation history of data.

Radial Error - A generalization of two horizontal error components (x, y) or three-dimensional (horizontal and vertical error components – x, y, z) error components to a distance value (magnitude) as measured along the radius of a circle or sphere, respectively.

Random Error - A category of error; a measure of deviation from an ideal or true value which results from an accidental and unknown combination of causes and varies from one measurement to the next. Not deterministic. For NSG applications, a random error is typically represented as a random variable, random vector, stationary process, or random field. And more specifically, as deviations about their mean-values, the latter considered biases. [b],[f]

- The random error corresponding to a random variable or the random error corresponding to (the elements of) a random vector is independent (uncorrelated) from one realization to the next, by definition.

- The random error corresponding to (the elements of) a random vector can also be correlated between the various elements for a given realization (intra-state vector correlation); hence this error is also a correlated error.
- The random error corresponding to a stochastic process corresponds to the collection of random errors associated with the collection of random vectors making up the stochastic process. Random error is independent (uncorrelated) from one realization to the next. However, within a specific realization, the individual random error vectors are typically temporally correlated amongst themselves (inter-state vector correlation); hence, random error is also correlated error. This same characteristic extends to random fields.
- The probability distribution for a random variable representing a random error is arbitrary – not necessarily Gaussian.

Random Field - A random field (RF) is a collection of random vectors (RV), parameterized by an N-dimensional spatial vector q. In general, two different random vectors from the same realization of the random field are correlated. In the NSG, when error is represented by a random field, its corresponding statistics are specified by a statistical error model. A general descriptor of a given random field is as follows: a ("scalar" or "multi-variate") ("homogeneous" or "non-homogeneous") "ND random field".

- Scalar (n=1) or multi-variate (n>1) refers to the number of elements n that each random vector contains and is sometimes described as "(nd)", e.g. (2d) corresponds to 2 elements (random variables) per random vector.
- Homogeneous or non-homogeneous refers to whether the corresponding statistics are invariant or vary over spatial location q.
- ND refers to the number of spatial dimensions (number of elements in q), e.g. 3D corresponds to 3 spatial dimensions. Each random vector corresponds to a unique value of q.
- As an example of terminology, "a multi-variate homogeneous 3D random field" or more specifically "a homogeneous 3D random field (2d)" corresponds to a multi-variate homogeneous random field over 3 spatial dimensions (q is a vector with 3 elements). The random vectors contain 2 elements.
- Spatial dimensions are general. For typical NSG applications, they correspond to some combination of geolocation directions and time. Note that a stochastic process is also a random field with N=1.
- In general, the collection of random vectors is infinite for a random field; however, only a finite subset is of interest for most applications, i.e., random vectors associated with a finite set of spatial locations.
- For typical NSG applications, the spatial correlation of a random field is specified by one of more strictly positive definite correlation functions (spdcf) contained in the corresponding statistical error model.

Random Variable - A variable whose value varies by chance, i.e., non-deterministic. Somewhat more formally, a random variable is a mapping from the space of experimental outcomes to a space of numbers.

In the NSG, when error is represented by a random variable (a random vector with one component or element, i.e., n=1), its corresponding statistics are specified by a statistical error model.

- For most NSG applications, the space of experimental outcomes is already a number. For example, the x-component of sensor position can be considered a random variable. Equivalently, it can be defined as the true x-component of sensor position plus x-component of sensor position error, the former a deterministic (typically unknown) value and the latter a random variable.
- A random variable is statistically characterized by its mean-value, variance, and (more completely) its probability density function (pdf). The probability density function (pdf) is typically unknown and not included, but if needed for the calculation of probabilities, assumed Gaussian distributed with the pdf completely characterized by the mean-value and variance.

Random Vector - A random vector (RV) is an nx1 vector which contains n random variables as components or elements. In the NSG, when error is represented as a random vector, its corresponding statistics are specified by a statistical error model. The corresponding random vector is also sometimes termed a random error vector.

- The realization of a Random Vector corresponds to a specific value of the vector (components or elements) for a given event such as a trial or experiment. Important descriptive statistics of a RV are its mean (vector) value and the error covariance matrix about the mean, and optionally, a multi-variate probability density function. These statistics can be predictive or sample-based.

Realization - For NSG accuracy and predicted accuracy applications, a specific trial or experimental outcome or independent sample involving a random error (category of error).

Relative Horizontal Accuracy - The range of values for the error in the difference between two objects' horizontal metric geolocation values with respect to a specified geodetic horizontal reference datum; e.g. expressed as a radial error at the 90 percent probability level (CE90). There are two types of relative horizontal accuracy: predicted relative horizontal accuracy is based on error propagation via a statistical error model(s); and measured relative horizontal accuracy is an empirically derived metric based on sample statistics.

Relative Vertical Accuracy - The range of values for the error in the difference between two objects' vertical metric geolocation values with respect to a specified geodetic vertical reference datum; e.g. expressed as a linear error at the 90 percent probability level (LE90). There are two types of relative vertical accuracy: predicted relative vertical accuracy is based on error propagation via a statistical error model(s); and measured relative vertical accuracy is an empirically derived metric based on sample statistics.

Rigorous Error Propagation - Represents the proper statistical modeling of all significant errors and their interrelationships throughout an NSG system. It enables optimal solutions as well as reliable predicted accuracies associated with specific estimates and products across the system modules.

Scalar Accuracy Metrics (augmented definition) - convenient one-number summaries of geolocation accuracy and geolocation predicted accuracy expressed as a probability: [b], [f], and [h]

- Linear Error (LE) - LE is an unsigned value that corresponds to the length of a vertical line (segment) such that there is a 90% probability that the absolute value of vertical error resides along the line. If the line is doubled in length and centered at the target solution, there is a 90% probability that the true target vertical location resides along the line. LE_XX corresponds to LE at the XX % probability level.
- Circular Error (CE) - CE is an unsigned value that corresponds to the radius of a circle such that there is a 90% probability that the horizontal error resides within the circle; or equivalently, if the circle is centered at the target solution, there is a 90% probability the true target horizontal location resides within the circle. CE_XX corresponds to CE at the XX % probability level.
- Spherical Error (SE) - SE is an unsigned value that corresponds to the radius of a sphere such that there is a 90% probability that 3d error resides within, or equivalently, if the sphere is centered at the target solution, there is a 90% probability that the true target location resides within the sphere. SE_XX corresponds to SE at the XX % probability level.

For the above scalar accuracy metrics:

- It is assumed that the underlying x-y-z coordinate system is a local tangent plane system, i.e., x and y are horizontal components and z the vertical component.
- CE-LE corresponds to the CE-LE error cylinder. There is a probability between 81 to 90 percent that 3d radial error resides within the cylinder. The former value corresponds to uncorrelated horizontal and vertical errors, the latter value to highly correlated horizontal and vertical errors.
- LE_XX, CE_XX, and SE_XX (aka LEXX, CEXX, and SEXX, respectively) are also called XX percentiles for absolute vertical errors, horizontal radial errors, and spherical radial errors, respectively. XX is expressed as an integer greater than zero and less than 100.

Sensor support data – See "metadata".

Spatial Correlation - The correlation between the elements (random variables) of two random vectors at two different spatial locations associated with the same realization of a random field.

Standard Deviation – The square root of the variance of a random variable. A measure of deviation or dispersion about the random variable's mean-value.

State Vector - A vector of parameters or variables that describe a system's state.

State Vector Error - A vector of errors corresponding to an estimate of a state vector relative to a (typically unknown) true state vector; a random vector of errors, or random error vector.

Stationary - A descriptor for a stochastic process with corresponding (*a priori*) statistics invariant over time. See homogeneous as well for random fields, which if corresponding to one spatial dimension are stochastic processes.

Stochastic Process - A stochastic process (SP) is a collection of random vectors (RV), parameterized by a 1D quantity, typically time. For a given realization of the stochastic process, the individual random vectors are correlated with each other. If the random vectors consist of one element or component (n=1), the stochastic process is sometimes called a scalar stochastic process, and if greater than one, a multi-variate stochastic process. A stochastic process is also a random field with one spatial (or time) dimension, i.e., N=1. In the NSG, when error is represented as a stochastic process, its corresponding statistics are specified by a statistical error model.

Strictly Positive Definite Correlation Function (spdcf) - A function which models the statistical correlation between random vectors (random variables), typically applied in the NSG to describe the temporal correlation and/or spatial correlation between various random vectors which are part of a stochastic process or random field, i.e., the spdcf is a function of delta time or delta distance (possibly in each of multiple directions) between random vectors. The proper use of an spdcf ensures assembly of a valid multi-state vector error covariance matrix, i.e., positive definite and symmetric.

Systematic Error - An error characteristic or error effect due to errors that are represented by random variables, random vectors, stochastic processes, or random fields. For example, an effect on observations (samples) such that their pattern of magnitude and direction are consistent but not necessarily constant. Such an effect can be associated with: [f], [j]

- Error(s) represented by a stochastic process or random field which appear systematic across time or space, respectively, due to temporal or spatial correlation, respectively.
- The error in a frame image-to-ground sensor model's adjustable parameter for focal length. This error is typically represented by a random variable, with a mean-value of zero and a constant variance, but its effect when projected to the ground appears as a systematic error across ground locations, e.g., it has a scaling effect which increases the closer the ground point to the image footprint's boundary.

Temporal Correlation - The correlation between the elements (random variables) of two random vectors at two different times associated with the same realization of a stochastic process.

Time Constant - The delta time value such that the correlation coefficient for temporal correlation, expressed as a decaying exponential function, equals $e^{-1} \cong 0.37$.

Uncertainty – A lack of certainty; limited knowledge; unknown or imperfect information. In terms of NSG applications, more general than the concepts of errors and accuracy, but sometimes used informally as a synonym. Applies to predicted accuracy but not to empirical (sample-based) accuracy.

Uncorrelated Error - At an intuitive level, an error that is statically unrelated to all other relevant errors. More precisely, if two random variables represent two uncorrelated errors (about their respective mean-values), their covariance and their corresponding correlation coefficient are zero. A random variable is uncorrelated (with itself) from one realization to the next, by definition. This latter property is also true for the random variables making up random vectors, stochastic processes, and random fields. However, these three representations typically include correlated errors within the same realization.

Uncorrelated Values - Values (of random variables or errors) which are statistically unrelated. [f] This is represented for two random variables by their covariance with a value of zero.

Vertical Error - As applied to geospatial measurements and processes, vertical error is a signed and one dimensional (linear) error value typically observed in the direction of the z-axis of a local right-handed coordinate system where the x, y plane is defined as tangent to the defined reference surface at the point of origin and the z-axis is normal to the x, y plane and positive in the up direction.

WGS 84 - World Geodetic System 1984 – A documented and formally maintained global coordinate system which allows an unambiguous representation of positional information by providing the basic reference frame (coordinate system), geometric figure for the earth (ellipsoid), earth gravitational model, and means to relate positions on various geodetic datums and systems for DoD operations and applications. [g]

Appendix B: A Variation of the Geolocation System: External Elevation

A common variation of the Geolocation System described in Section 4.6.1 of the main body of this document corresponds to the use of single image extractions (aka monoscopic extraction) instead of stereo image extractions. Such a system is illustrated in Figure B-1, a repeat of Figure 4.6.1-1 for easier context.

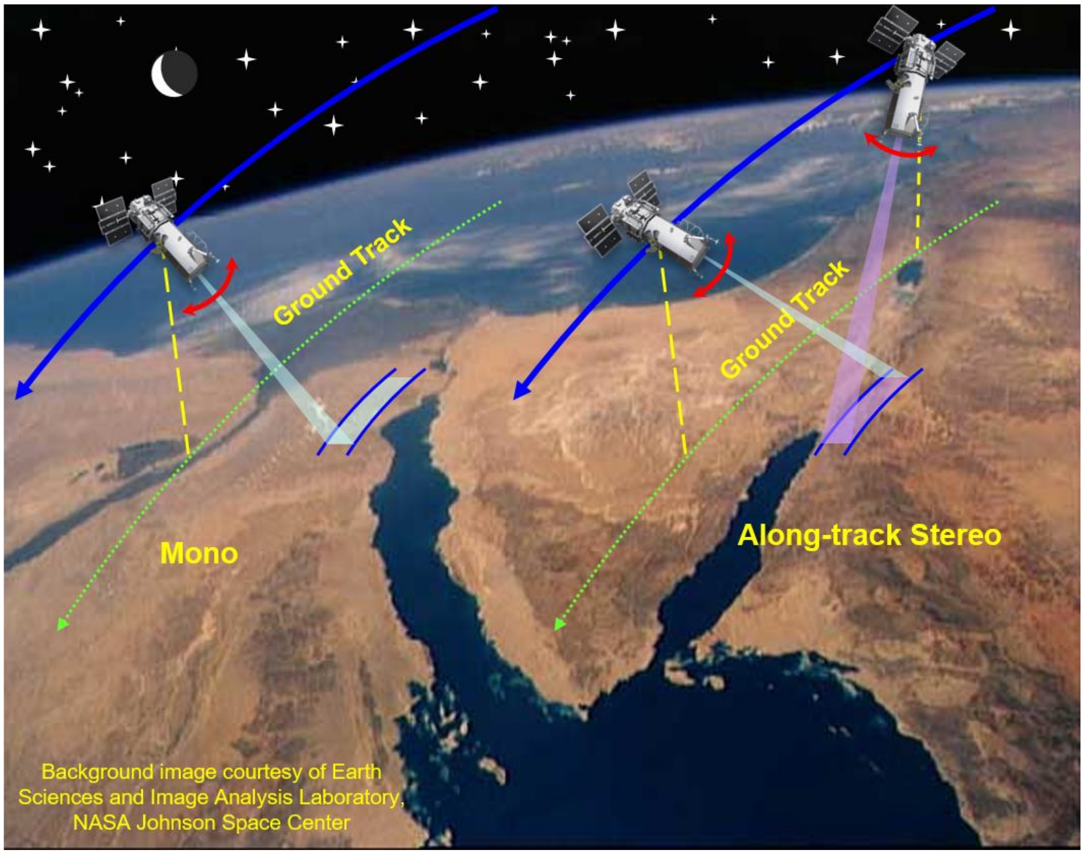

Figure B-1: Example of a Single-image (Mono) EO Imaging System – left side of the above graphic

Although this variation is applicable to a specific Geolocation System, a commercial satellite-based imaging system, in particular, its underlying principles and approach are applicable to many other systems that do not utilize an imaging sensor. The common characteristic is the use of a single sensor-based measurement that is inherently 2d in order to extract a 3d geolocation, and the corresponding need for additional information – an external estimate of elevation or height as detailed below.

In the single image-based system, a 2d measurement (line, sample) of the location of a 3d geospatial object of interest from an image does not provide enough information; thus, an *a priori* estimate of its corresponding elevation must also be provided, such as from a Digital Elevation Model (DEM) or Digital Surface Model (DSM), in order to extract the 3d location. For a specific location, this is typically accomplished using MIG, with inputs consisting of the image measurement and the *a priori* elevation estimate.

The MIG's output consists of the estimator solution, basically the intersection of image-to-ground line-of-sight (LOS) vector with the DEM, along with the solution's predicted accuracy. The image-to ground LOS is based on the image measurement, the image metadata, and the sensor image-to-ground function. And in this case, the term "MIG" is a misnomer as it really is based on only one image, and is sometime replaced by the term "SIG" corresponding to single image. The solution is also accompanied by predicted accuracy, a function of the various errors, including sensor metadata errors and DEM errors. Correspondingly, system accuracy for an arbitrary solution or extraction is typically equal to the predicted accuracy for a representative geolocation, and typically at an elevation angle at a lower value within the sensor-to-ground operational range, since the lower the value of the elevation angle the larger the corresponding effect of elevation error on horizontal error.

Accuracy is then defined for an arbitrary monoscopic extraction the same as for an arbitrary stereo extraction as detailed earlier, with one exception: There are two choices for the specified LE90, which are listed below and illustrated in Figure B-2:

5) LE90 is set equal to the accuracy of the DEM assumed available to the Geolocation System, with CE90 set equal to the appropriate horizontal accuracy due to both the normal extraction errors, typically dominated by the sensor metadata errors, and the effect of the DEM elevation errors on the horizontal errors – see Figure B-2. This effect increases as the elevation angle decreases.
6) LE90 is set equal to a negligible value, with CE90 set appropriately for horizontal accuracy with no effect due to elevation errors – see the upper right portion of Figure B-2. Any corresponding specification of Geolocation System accuracy explicitly states this assumption, which allows those interested to inflate CE90 appropriately based on the accuracy of the assumed elevation that they will be able to access.

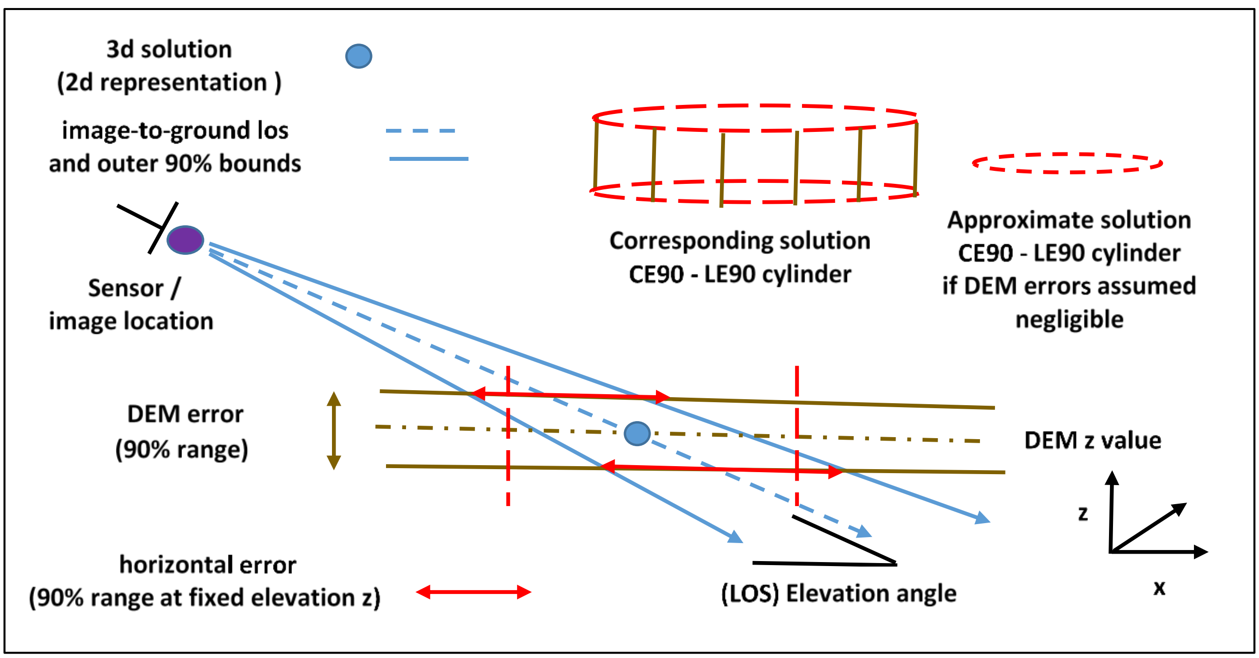

Figure B-2: The effect of elevation errors on horizontal extraction errors assuming a single EO image extraction with DEM; a function of LOS elevation angle; figure not to scale.

In Figure B-2 the sensor LOS 90% outer-bounds are due to the combined effects of sensor metadata errors: position and attitude (sensor pose) errors as well as errors in any calibration corrections, such as focal length correction. Also, because this is an EO scanning sensor, the LOS is virtually aligned with the imaging locus, the blue dotted line in the figure. This locus is more generally termed the “geolocation locus” in order to encompass non-imaging sensors as well. The geolocation locus is defined as corresponding to all possible geolocations that are consistent with the sensor measurement, a 2d image pixel (line, sample) in the above example. The effect of elevation errors on horizontal errors is more generally a function of the elevation angle relative to the geolocation locus in the local tangent plane.

Finally, as mentioned earlier, the above paradigm regarding use of an external elevation for necessary additional information is also applicable to sensors other than imaging sensors, i.e., those sensors with corresponding 2d measurements.

Appendix C: A Variation of the Geolocation System: Sensor Metadata – Representation and Specification of Accuracy and Predicted Accuracy

Similar concepts of accuracy and predicted accuracy and the corresponding statistical error model are applicable to the other “up-stream” NSG modules, such as the Collection and the Value-Added Processing modules, and not just the Exploitation module and explicit geolocations as were discussed in Section 4.6.1 of the main document and in Appendix B. Correspondingly, the accuracy and predicted accuracy typically do not correspond to 3d geolocation errors, but to the errors in other relevant objects or state vectors, such as sensor metadata.

For example, sensor metadata typically corresponds to an estimate of a n component state vector containing sensor pose (position and attitude), possibly sensor calibration parameters, etc. This estimate is generated by an estimator within the NSG Geolocations System’s Collection Module and possibly further refined by its Value-Added Module. The estimate’s predicted accuracy consists of an n×n error covariance matrix with an assumed mean-value of error equal to zero, contained as part of its statistical error model. The error in the estimate corresponds to specific sensor metadata and the predicted accuracy provides a statistical description of this unknown error via the statistical error model.

On the other hand, system accuracy, as opposed to predicted accuracy, typically corresponds to accuracy requirements for the estimator in general that resides within the Collection Module and/or Value-Added Module, i.e., applicable to the errors in an arbitrary state vector estimate of various applicable sensor metadata. It is specified by appropriate statistical metrics and/or probabilistic values for various sub-collections of the n components of sensor metadata error, or a metric that is a function of these components, similar to CE90 and LE90 for geolocation error.

There are three general methods that can be used to specify sensor metadata system accuracy: Geolocation Equivalent, State Vector Direct, and Sensor Direct. The latter is the most “straight-forward” and illustrated as follows:

<u>Sensor Direct</u>

A representative example of the Sensor Direct method is presented in Figure C-1. It specifies system accuracy as it directly relates to an EO imaging sensor and a measurement of geolocation from that sensor.

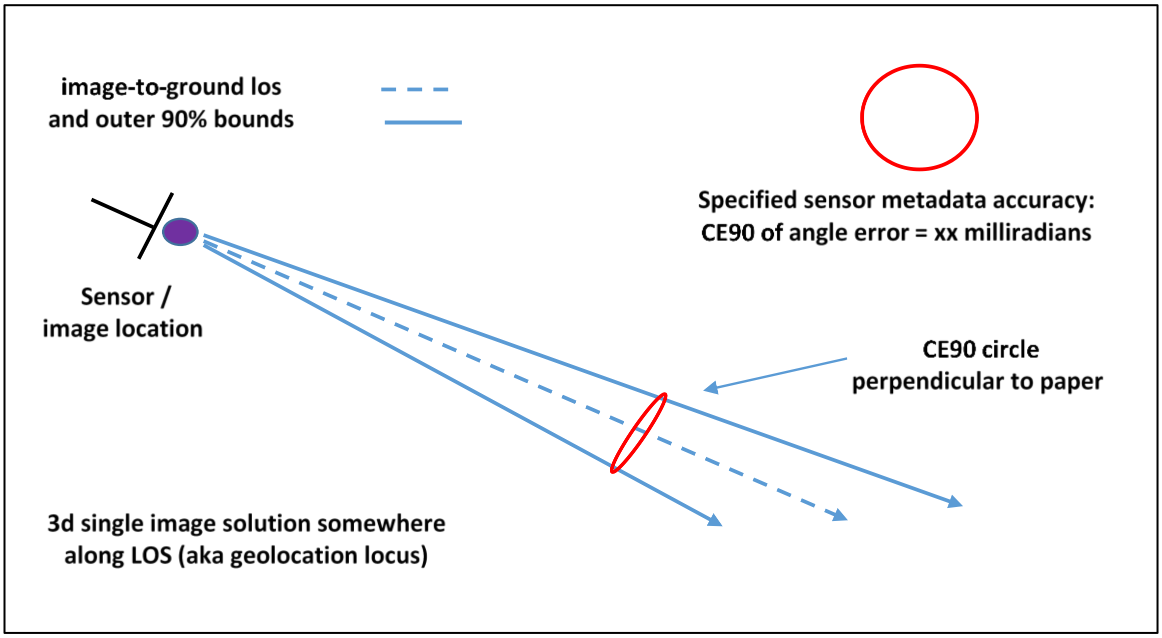

Figure C-1: Sensor Direct representation of sensor metadata accuracy corresponding to EO imagery

In this particular example, the sensor direct method specifies sensor metadata accuracy as CE90 angular error along the geolocation locus, or the sensor-to-ground line-of-sight vector since an EO imaging sensor. This is similar to the method detailed in [1, pg. 34]. Note also that this method is “stand alone” in that it requires no assumption regarding external data, such as an elevation or height of a geolocation and its assumed accuracy. And as a reminder, we are discussing system accuracy, not the predicted accuracy associated with a specific geolocation object.

Another example of the Sensor Direct method corresponds to SAR imagery. Figure C-2 presents a general comparison between EO and SAR imagery as background information, with [13] a general reference for SAR and related imagery. The corresponding Sensor Direct method for SAR imagery is presented Figure C-3 and is different in content from that for EO imagery.

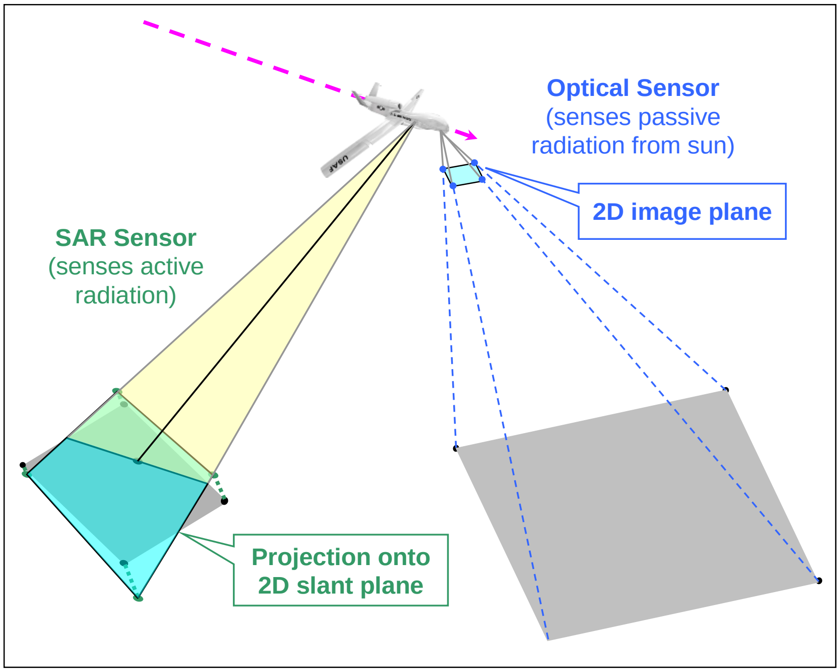

Figure C-2: Overview of SAR and optical sensors; originally from [3]

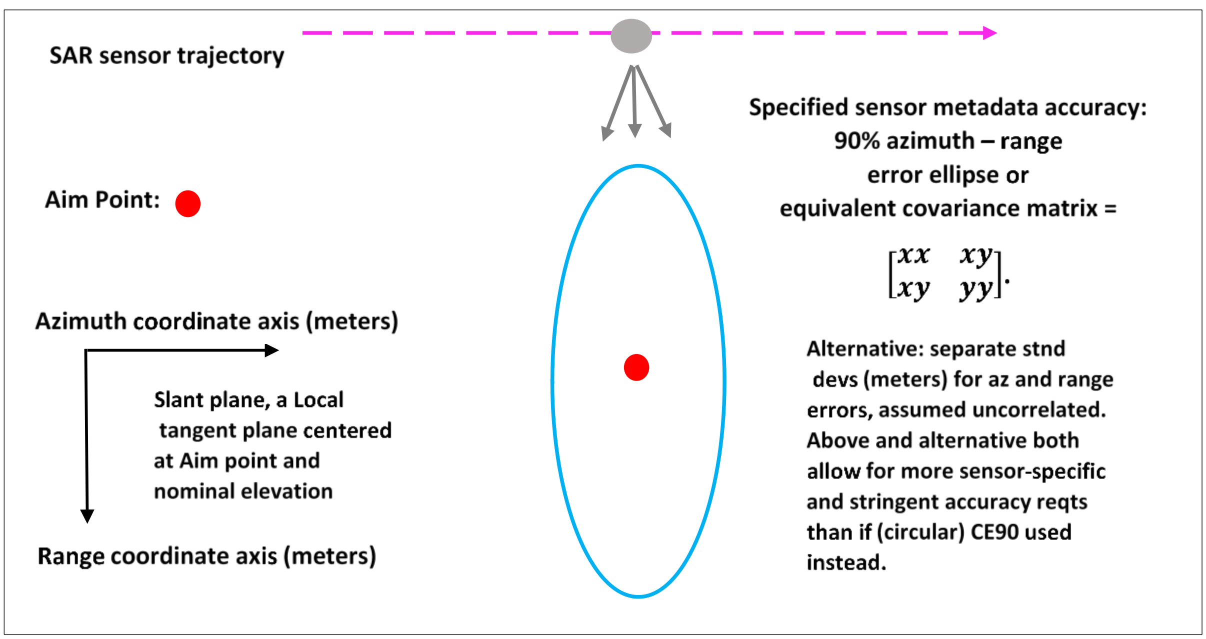

Figure C-3: Sensor Direct representation of sensor metadata accuracy corresponding to SAR imagery; the specification of the 90% azimuth – range error ellipse.

Some interesting features of SAR sensor metadata and imagery that are in complimentary contrast to EO sensor metadata and images that are not explicitly illustrated in the above figures are as follows:

1) SAR sensor metadata includes sensor velocity, not sensor attitude;
2) the SAR geolocation locus corresponds to a circle at the base of a range-doppler cone, not a line;
3) if an elevation from a DEM or DSM intersects the SAR geolocation locus, the elevation error's contribution to horizontal error decreases with decreasing LOS elevation, not increases, i.e., measurements from SAR and EO imagery are complimentary.

Finally, note that the Sensor Direct method for the representation of sensor metadata (system) accuracy is also sometimes applied for the representation of sensor metadata predicted accuracy for specific sensor metadata. An example corresponds to sensor metadata represented using a Rational Polynomial Coefficient (RPC) sensor model, as documented in [9].

Summary of all three General Methods for the specification of sensor metadata accuracy

As mentioned earlier there are three general methods to specify system sensor metadata accuracy: Geolocation Equivalent, State Vector Direct, and Sensor Direct. They are summarized in Table C-1.

Table C-1: Methods for the specification of system accuracy for sensor metadata

Method	Approach	Comments
Geolocation Equivalent	Specify as geolocation accuracy, CE90 for horizontal errors and LE90 for vertical or elevations errors, assuming sensor metadata errors are the only errors affecting the corresponding geolocation solutions or extractions. A specification for sensor metadata accuracy in a Geolocation System: geolocation errors due to sensor metadata errors correspond to CE90 <= xx meters and LE90 <= yy meters. The above errors may also include the contirbution of a nominal amount of sensor mensuration (measurement) error as well if specifically specified.	Convenient and expressed directly as corresponding effect on geolocations. Representative sensor metadata error covariance matrices can be input to a representive WLS solution (e.g., MIG solution, if images), and the solution covariance matrix used to generate CE90 and LE90 used to specify sensor metadata accuracy for an arbitrary sensor metadata. The WLS solution can also use multiple measurements and *a priori* elevation if applicable to the Geolocation System's operational scenario.
State Vector Direct	Specify as expected magnitude of various subgroups of state vector component errors, either as rms, maximum standard deviation, or as scalar accuracy metrics, such as LE90 , CE90 ,or SE90. Example of the specified accuracy of arbitrary sensor metadata corresponding a Geolocation System based on an EO imaging sensor(s): sensor 3d postion errors SE90 <= xx1 meters, sensor 2d rotation angle errors about the image plane axes CE90 <= xx2 milliradians, sensor 1d rotation angle errors about the focal lenth axis LE90 <= xx3 milliradians, and focal length correction errors LE90 <= xx4 micrometers.	Directly applicable to any sensor and corresponding sensor metadata. Group similar components with similar units and express corresponding statistical metrics using those units. Expected value of all errors assumed zero unless specifically stated (statistically bounded) otherwise. Statistical metrics have appropirate units; for example if a subgroup corresponds to attitude, applicable units may be milliradians.

Table C-1 (continued): Methods for the specification of system accuracy for sensor metadata

Method	Approach	Comments
Sensor Direct	Specify sensor metadata accuracy as statistical metrics, such as standard deviations or scalar accuracy metrics, LE90, CE90, or SE90, as appropriate. The corresponding errors are either: (1) the projection of sensor metadata errors perpendicular to the geolocation locus based on a single sensor measurement, or (2) the projection of sensor metadata errors to geolocations based on either one sensor measuement if it is inherently 3d (e.g., LiDAR) or one sensor measurement and the assumed and specified use of an elevation.	Directly associated with the sensor, its metadata and type of measurement. The geolocation locus is defined as all possible geolocations consistent with the sensor measurement. Examples: (1) Figure C-1 corresponding to the geolocation locus assuming one EO image and corresponding measurement: accuracy expressed as CE90 angular error; (2) Figure C-3 corresponding to a geolocation assuming one SAR image and corresponding measurement and a a known elevation: accuracy expressed as 90% error ellipse corresponding to range and azimuth horizontal ground coordinates for a SAR sensor.

Appendix D: Additional Comments Regarding Specification and Validation

This appendix presents additional comments regarding Sections 5.1, 5.1.1, and 5.1.2 in the main body of this document for a more complete overview of specification and validation of accuracy and predicted accuracy requirements.

<u>Section 5.1.1 and 5.1.2 – Examples of Specification and Validation</u>

1. Both the specifications for accuracy and predicted accuracy requirements have an associated operational range for the extraction of geolocations associated with the Geolocation System. For example, the range of applicable imaging angles if geolocations are extracted from EO images. If the operational range is too large (varied), specifications for sub-categories of the range are applicable.

2. TGD 2c also defines and discusses Type I and Type II validation errors for both accuracy and predicted accuracy. A Type I error corresponds to validation fails when it should have passed, and a Type II error corresponds to validation passes when it should have failed. The probabilities of validation errors are quantified as a function of various combinations of relevant parameters: the number of samples n, and the specified probability levels XX, confidence levels YY, and/or level of predicted accuracy fidelity.
3. The predicted mean-value for errors is assumed equal to zero, as is usual.
4. The 100 samples of horizontal radial error contained in Figure 5.1.2-3, "radial errors versus predicted 90% radials", demonstrate variation due to inherent variation of underlying horizontal errors, regardless whether the latter's error covariance matrices are the same or not. That is, even if all of the corresponding error covariance matrices were identical and, correspondingly, independent samples considered associated with only one error covariance matrix, samples of horizontal error would vary inherently consistent with an approximate Gaussian probability distribution of error characterized by a mean-value of zero and the specified covariance matrix.
5. All error samples in the examples presented in these sections were simulated per the Monte-Carlo simulation technique for random vectors summarized in Section 5.11.1 of this document.
6. Successful validation of predicted accuracy requirements ensures that geolocation extractions include reliable error covariance matrices. As such, reliable scalar accuracy metrics (e.g., CE90) are also ensured if optionally computed from the error covariance matrices.
7. The specification and validation of predicted accuracy can also be performed using individual values of scalar accuracy metrics (e.g., $CE90_i$), instead of their error covariance matrix counterparts $C_{\epsilon X_i}$, if the former are available, but the latter are not. This is not the preferred approach, but is detailed in TGD 2c for completeness.

Section 5.1 – Overview and General Functional Flow

1. Although Section 5.1 addresses specification and validation explicitly, the methods and algorithms presented are also applicable to general assessments of accuracy and predicted accuracy for a Geolocation System.
2. Although not illustrated explicitly in Figure 5.1-1, Standard Application Program Interfaces (APIs) are recommended for communication between modules, both operationally and for validation/verification; in particular, those associated with state (vector) error models. This standardization helps to ensure compatibility and efficiency between and within various NSG modules.
3. In Figure 5.1-1, the relative error between two different estimates of geolocations, X_i and X_j, is defined as $\epsilon X_{ij} = \epsilon X_i - \epsilon X_j$, with corresponding error covariance matrix $relC_{\epsilon X_{ij}} = C_{\epsilon X_i} + C_{\epsilon X_j} - C_{\epsilon X_{ij}} - C_{\epsilon X_{ji}}$, where $C_{\epsilon X_{ij}}$ is the cross-covariance matrix relative to the two errors. These two errors are expected to be correlated ($C_{\epsilon X_{ij}} \neq 0$) if, for example, corresponding sensor metadata is common or temporally correlated. If so, and in order to assess predicted relative accuracy, either: (1) the geolocations are estimated together in an expanded X_i and their cross-covariance $C_{\epsilon X_{ij}}$ contained in the corresponding (expanded) $C_{\epsilon X_i}$, or (2) the geolocations continue

to be estimated as different X_i and X_j and their cross-covariance $C_{\epsilon X_{ij}}$ is computed in a separate process as described in TGD 2d (Estimators and their QC).

4. As illustrated in Figure 5.1-1, the Value-Added Processing module may be bypassed in order to test exploitation results due to the Collection module only. In either case, exploitation is actually based on a Trusted Exploitation Application, instead of the actual Exploitation module, for independence of the validation process. Additional verification (as opposed to validation) tests can be performed by comparing outputs from the Trusted Exploitation Application with corresponding outputs from the Exploitation module. In addition, for some Exploitation modules, the representative state vector X and its corresponding X_i contained within, need not correspond to explicit geographic locations, but to any well-defined state vector with corresponding "ground" truth available.